Finals Study Guide – Day 3 of 4: Simple Harmonic Motion & Waves

\[ T = 2\pi \sqrt{\frac{L}{g}} \]  
Period of a Pendulum = \(2\cdot \pi \cdot \sqrt{\frac{\text{Length of Pendulum}}{\text{Acceleration due to gravity}}}\)

\[ T = 2\pi \sqrt{\frac{m}{k}} \]  
Period of a Mass-Spring = \(2\cdot \pi \cdot \sqrt{\frac{\text{Mass}}{\text{Spring Constant}}}\)

\[ f = \frac{1}{T} \]  
Frequency = \(\frac{1}{\text{Period}}\)

\[ T = \frac{1}{f} \]  
Period = \(\frac{1}{\text{Frequency}}\)

\[ v = \lambda \cdot f \]  
Wave speed = Wavelength \cdot Frequency

\[ f_{\text{beat}} = |f_1 - f_2| \]  
Beat Frequency = \(|\text{Frequency}_1 - \text{Frequency}_2|\)

\[ f_n = n \cdot f_1 \]  
Frequency of \(n^{\text{th}}\) harmonic = \(n \cdot \text{Fundamental Frequency}\)

\[ v_{\text{air}} = 343 \text{ m/s} \text{ (assumed 20 °C non-humid air)} \]

\[ c = 3 \times 10^8 \text{ m/s} \]

1. You need to know the height of a tower, but darkness obscures the ceiling. You note that a pendulum extending from the ceiling almost touches the floor and that its period is 24 seconds. How tall is the tower?

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 24\text{ s})</td>
<td>(L = ?)</td>
<td>(T = 2\pi \sqrt{\frac{L}{g}})</td>
</tr>
</tbody>
</table>

\[ 24 = 2(3.14) \sqrt{\frac{L}{9.8}} \]  
\[ (3.82)^2 = \left(\sqrt{\frac{L}{9.8}}\right)^2 \]

\[ L = 143 \text{ m} \]

\[ \frac{24}{6.28} = \frac{9.8 \times 14.59}{9.8} = \frac{L}{9.8} \times 9.8 \]

2. A trapeze artist swings in simple harmonic motion with a period of 3.8 s. Calculate the length of the cables supporting the trapeze. What is the frequency of the trapeze artist?

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<tr>
<td>(T = 3.8\text{ s})</td>
<td>(L = ?)</td>
<td>(T = 2\pi \sqrt{\frac{L}{g}})</td>
</tr>
</tbody>
</table>

\[ 3.8 = 2(3.14) \sqrt{\frac{L}{9.8}} \]  
\[ (0.605)^2 = \left(\sqrt{\frac{L}{9.8}}\right)^2 \]

\[ 3.8 = 2(3.14) \sqrt{\frac{L}{9.8}} \]

\[ 9.8 \times 0.366 = \frac{L}{9.8} \times 9.8 \]

\[ L = 3.58 \text{ m} \]

<table>
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<td>(f = \frac{1}{T})</td>
<td>(f = \frac{1}{3.8} = 0.263 \text{ Hz} )</td>
</tr>
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</table>
3. Calculate the period and frequency of a 3.5 m long pendulum at the North Pole where gravity is 9.832 m/s².

**Knowns**
- \( L = 3.5 \text{ m} \)
- \( g' = 9.832 \text{ m/s}^2 \)

**Unknowns**
- \( T = ? \)
- \( f = ? \)

**Formula**
- \( T = 2\pi \sqrt{\frac{L}{g'}} \)
- \( f = \frac{1}{T} \)

\[ T = 3.95 \text{ s} \]
\[ f = \frac{1}{T} = \frac{1}{3.95} = 0.256 \text{ Hz} \]

4. A 125 N object vibrates with a period of 3.56 s when hanging from a spring. What is the spring constant of the spring?

**Knowns**
- \( W_t = 125 \text{ N} \)
- \( T = 3.56 \text{ s} \)
- \( g' = 9.8 \text{ m/s}^2 \)

**Unknowns**
- \( k = ? \)

**Formula**
- \( T = 2\pi \sqrt{\frac{m}{k}} \)
- \( W_t = mg' \)

\[ 3.56 = 2\pi \sqrt{\frac{125}{k}} \]
\[ 3.56 = 2(3.14) \sqrt{\frac{125}{k}} \]
\[ 0.566 = \left(\frac{\sqrt{125}}{k}\right)^2 \]
\[ 0.32 = \frac{12.755}{k} \]

\[ k = 39.86 \text{ N/m} \]

5. The body of a 1275 kg car is supported on a frame by four springs. Two people riding in the car have a combined mass of 153 kg. When driven over a pothole in the road, the frame vibrates with a period of 0.840 s. Find the spring constant of a single spring.

a. When two more people get into the car, the total mass of all four occupants of the car becomes 255 kg. What is the new period of vibration?

**Knowns**
- \( m_{\text{Car}} = 1275 \text{ kg} \)
- \( m_{\text{ ppl}} = 153 \text{ kg} \)
- \( T_{\text{ original}} = 0.84 \text{ s} \)

**Unknowns**
- \( K = ? \)

**Formula**
- \( T = 2\pi \sqrt{\frac{m}{4k}} \)
- \( m_{\text{Total}} = 255 \text{ kg} + 1275 \text{ kg} = 1530 \text{ kg} \), \( \frac{1530}{4} = 382.5 \text{ kg/spring} \)

\[ 0.84 = 2(3.14) \sqrt{\frac{382.5}{K}} \]
\[ 0.84 = 6.28 \sqrt{\frac{382.5}{K}} \]

\[ 0.1338 = \left(\frac{\sqrt{382.5}}{K}\right)^2 \]
\[ 0.0179 = \frac{382.5}{K} \]

\[ K = \frac{382.5}{0.0179} = 21,300 \text{ N/m} \]

**New period**

\[ T = 2\pi \sqrt{\frac{382.5}{19944}} \]
\[ T = 6.28 \sqrt{0.01918} \]

\[ T = 0.875 \text{ s} \]
6. A piano emits frequencies that range from a low of about 28 Hz to a high of about 4200 Hz. Find the range of wavelengths in air attained by this instrument when the speed of sound in air is 343 m/s.

\[ \frac{343 = 28 \lambda}{28} \quad \lambda_L = 12.25 \text{ m} \]

\[ \frac{343 = 4200 \lambda}{4200} \quad \lambda_H = 0.0816 \text{ m} \]

7. The red light emitted by a He-Ne laser has a wavelength of 633 nm in air and travels at 3x10^8 m/s. Find the frequency of the laser light. (1 nm = 1x10^-9 m)

\[ \frac{3 \times 10^8 = (633 \times 10^{-9}) f}{633 \times 10^{-9}} \quad f = 4.739 \times 10^{14} \text{ Hz} \]

8. A tuning fork produces a sound with a frequency of 256 Hz and a wavelength in air of 1.35 m.

a. What value does this give for the speed of sound in air?

b. What would be the wavelength of this same sound in water in which sound travels at 1500 m/s?

\[ \frac{v = \lambda f}{256} = 1.35 \cdot 256 = 345.6 \text{ m/s} \]

\[ \frac{\lambda = \frac{1500}{256} = \lambda \cdot 256}{256} \quad \lambda = 5.859 \text{ m} \]
9. On a piano, the note middle C has a fundamental frequency of 262 Hz. What is the fourth harmonic of this note?

\[ f_1 = 262 \text{ Hz} \]

\[ f_4 = 4 \cdot 262 \text{ Hz} = 1048 \text{ Hz} \]

10. A piano tuner using a 392 Hz tuning fork to tune the wire for G-natural hears four beats per second. What are the two possible frequencies of vibration of this piano wire?

a. The piano tuner then tightens the wire, strikes the key with the 392 Hz fork and now hears two beats per second. Is the piano tuner getting closer to successfully tuning the piano or not?

\[ f = 392 \text{ Hz} \]

\[ f_{\text{beat}} = 4 \]

\[ 4 = \left| 392 - f \right| \]

\[ 4 = 392 - f \]

\[ f = 388 \text{ Hz} \] and \[ f = 396 \text{ Hz} \]

a. Getting closer.

11. Two saxophones are being tuned during band. One saxophone plays a note with a wavelength of 0.6049 m while the other plays with a wavelength of 0.6352 m.

a. What are the frequencies of these sounds?

b. How many beats are being produced?

\[ \lambda_1 = 0.6049 \text{ m} \]

\[ \lambda_2 = 0.6352 \text{ m} \]

\[ v = 343 \text{ m/s} \]

\[ \frac{343}{0.6049} \]

\[ \frac{343}{0.6352} \]

\[ f_1 = 567 \text{ Hz} \]

\[ f_2 = 540 \text{ Hz} \]

\[ f_{\text{beat}} = \left| 567 - 540 \right| = 27 \text{ Hz} \]