Unit 8
Chapter 6: Triangle Trigonometry
6.1 Part 1: Area

Objectives: 1) Derive the area formula of a triangle; 2) Solve area problems

Homework: p. 410 #35 – 40

DO NOW:

![Diagram of a triangle with altitude drawn from point C to the x-axis labeled as 'h'.]

a. Label the altitude drawn from point C to the x-axis as 'h'.

b. What is \( \sin \theta \), in terms of line segments in the picture above?

\[
\sin \theta = \frac{h}{b}
\]

c. What is the height of this triangle, in terms of \( \sin \theta \) and \( b \)?

\[ h = b \sin \theta \]

NOTES: Area of any triangle:

\[
A = \frac{1}{2} bh
\]

\[
A = \frac{1}{2} (c \sin \theta)
\]

so \[ A = \frac{1}{2} \text{(base)} \times b \sin \theta \]

\( \theta \): angle between base + b.
Group Work:

**Example 1.** Jack is planting a triangular rose garden. The lengths of two sides of the plot are 8 feet and 12 feet, and the angle between them is 87°. Which expression could be used to find the area of this garden?

\[
\begin{align*}
(1) & \ 8 \cdot 12 \cdot \sin 87^\circ \\
(2) & \ 8 \cdot 12 \cdot \cos 87^\circ \\
(3) & \ \frac{1}{2} \cdot 8 \cdot 12 \cdot \cos 87^\circ \\
(4) & \ \frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 87^\circ \\
\end{align*}
\]

\[
\frac{1}{2} (8 \times 12 \times \sin 87^\circ)
\]

**Example 2.** The accompanying diagram shows the floor plan for a kitchen. The owners plan to carpet all of the kitchen except the “work space,” which is represented by scalene triangle \(ABC\). Find the area of this work space to the nearest tenth of a square foot.

\[
A = \frac{1}{2} (12 \times 31 \times \sin 62^\circ)
\]

\[
A = 164.2 \text{ ft}^2
\]

**Example 3.** Two sides of a triangular-shaped pool measure 16 feet and 21 feet, and the included angle measures 58°. What is the area, to the nearest tenth of a square foot, of a nylon cover that would exactly cover the surface of the pool?

\[
A = \frac{1}{2} (16 \times 21 \times \sin 58^\circ)
\]

\[
A = 142.5 \text{ ft}^2
\]
Example 4. The triangular top of a table has two sides of 14 inches and 16 inches, and the angle between the sides is $30^\circ$. Find the area of the tabletop, in square inches.

\[ A = \frac{1}{2} (14 \times 16) \sin 30^\circ \]

\[ A = 56 \text{ in}^2 \]

Example 5. In $\triangle ABC$, $AC = 18$, $BC = 10$, and $\cos C = \frac{1}{2}$. Find the area of $\triangle ABC$ to the nearest tenth of a square unit.

\[ \cos^{-1} \left( \frac{1}{2} \right) = 60^\circ \]
\[ C = 60^\circ \]

\[ A = \frac{1}{2} (18 \times 10 \sin 60^\circ) \]

\[ A = 77.9 \text{ units}^2 \]

Example 6. A landscape architect is designing a triangular garden to fit in the corner of a lot. The corner of the lot forms an angle of $70^\circ$, and the sides of the garden including this angle are to be 11 feet and 13 feet, respectively. Find, to the nearest integer, the number of square feet in the area of the garden.

\[ A = \frac{1}{2} (11 \times 13 \sin 70^\circ) \]

\[ A = 97 \text{ ft}^2 \]
Example 7. Gregory wants to build a garden in the shape of an isosceles triangle with one of the congruent sides equal to 12 yards. If the area of his garden will be 55 square yards, find, to the nearest tenth of a degree, the three angles of the triangle.

\[ \frac{55}{72} = \sin \gamma \]
\[ \sin^{-1} \left( \frac{55}{72} \right) = \gamma \]
\[ \gamma = 49.8^\circ \]

Example 8. The accompanying diagram shows a triangular plot of land that is part of Fran's garden. She needs to change the dimensions of this part of the garden, but she wants the area to stay the same. She increases the length of side AC to 22.5 feet. If angle A remains the same, by how many feet should side AB be decreased to make the area of the new triangular plot of land the same as the current one?

\[ A = \frac{1}{2} (22.5) (18) (\sin 36.87^\circ) \]
\[ A = 108 \text{ ft}^2 \]

Example 9. A garden in the shape of an equilateral triangle has sides whose lengths are 10 meters. What is the area of the garden, in exact form?

\[ A = \frac{1}{2} (10)(10)(\sin 60^\circ) \]
\[ A = 50 \cdot \frac{\sqrt{3}}{2} \]
\[ A = 25\sqrt{3} \text{ m}^2 \]