Unit 7 Review - Radicals and Rational Expressions

1. Simplify $8^{4/3}$.
   
   a. $\frac{1}{2}$
   
   b. $8 = 2^4 = 16$
   
   c. $\frac{32}{3}$

2. True or False:
   
   a. For any integer $a \neq 0$, $a^{1/n} = \frac{1}{a^n}$.  \textbf{False}
   
   b. For any integer $n > 0$ and any positive real number $a$, $a^{1/n} = \sqrt[n]{a}$. \textbf{True}

3. The volume of a dodecahedron (a solid with 12 regular pentagons as faces) is $V \approx 7.66312a^3$, where $a$ is the length of an edge. Find the edge length of a dodecahedron whose volume is 1000 cubic centimeters.

   $\frac{1000}{7.66312} = a^3 \Rightarrow a = \sqrt[3]{\frac{1000}{7.66312}} \approx 5.07 \text{ cm}$

4. Find values for $a$, $b$, and $c$ such that $27^{-2/3} = ((27)^a)^b$. Then simplify the expression $27^{-2/3}$.

   $27^{-2/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$

5. Simplify the expression.

   a. $\frac{1}{3} \sqrt[3]{128} = \frac{1}{3} \cdot 4 = \frac{4}{3}$
   
   b. $\frac{3}{2} \sqrt[3]{1024} = \frac{3}{2} \cdot 4 = \frac{6}{2} = 3$

6. Simplify the expression. Assume all variables are positive.

   a. $\sqrt[3]{18x^8y^9} = \sqrt[3]{2} \cdot \sqrt[3]{3} \cdot \sqrt[3]{4} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y} \cdot \sqrt[3]{y} = \sqrt[3]{2} \cdot \sqrt[3]{3} \cdot 8 \cdot x \cdot y \cdot y$
   
   b. $\sqrt[3]{6x^3y^7} \cdot \sqrt[3]{4x^5} = \sqrt[3]{6 \cdot 4} \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y = 2x^2y^2 \cdot \sqrt[3]{3x^2y}$

9. The area of a circular park is $2 \times 10^4$ square feet. The radius $r$ (in feet) can be approximated by the model

   $r = \left(\frac{4}{\pi}\right)^{1/2}$ where $A$ is the area of the park (in square feet). Approximate the radius of the park. Use 3.14 for $\pi$. Round your answer to the nearest whole number.

   $r = \left(\frac{2 \times 10^4}{\pi}\right)^{1/2} \approx 79.17 \approx 80$ ft.

10. Let $r(x) = 2x$ and $s(x) = x^3 - 3$. Find $s(r(2))$.

   a. $s(r(2)) = s(2) = 2^3 - 3 = 5$
   
   b. $s(r(2)) = s(2) = 4^3 - 3 = 61$

11. A discount function $D(x)$ that takes 10% off an entire purchase can be given by $D(x) = 0.90x$ where $x$ is the amount of the entire purchase. A tax function $T(x)$ that adds a tax of 10% to an entire purchase can be given by $T(x) = 1.10x$. Explain what $D(T(x))$ and $T(D(x))$ represent. Then compare the two compositions.

   a. $D(T(x))$ represents taking a 10% discount, then taking a 10% discount, then taxing the item.

   b. $T(D(x))$ represents taking a 10% discount, then taxing the item, then taking a 10% discount.

   The compositions are equal.
Let \( f(x) = (x - 3)^2 \) and \( g(x) = \frac{2}{x} \). Perform the indicated operation and state the domain.

12. \( f(g(x)) \)  
   \[ f\left(\frac{2}{x}\right) = \left(\frac{2}{x} - 3\right)^2 \]  
   Domain: \( x \neq 0 \)

13. \( g(f(x)) \)  
   \[ g((x-3)^2) = \frac{2}{(x-3)^2} \]  
   Domain: \( x \neq 3 \)

14. \( g(g(x)) \)  
   \[ g\left(\frac{2}{x}\right) = \frac{-2}{x} = -2 \cdot \frac{1}{x} = x \]  
   Domain: \( x \neq 0 \)

15. Find the inverse of the relation \((-5, 8), (8, -5), (-6, 1)\).  
   Inverse: \( x \leftrightarrow y \)  
   \( (8, -5), (1, -6) \)

16. Write an equation for the inverse of the relation \( y = -11x + 9 \).

17. Sketch the graph of the function \( f(x) = -x^2 - 1 \). Is the inverse of \( f(x) \) a function?

18. Explain how the inverse of a function is determined algebraically. Give an example to explain the steps you follow.  
   switch \( x \) and \( y \), then solve for \( y \). Look at \#16.

19. Graph the function. Then state the domain and range.

20. \( \frac{x^{3d}}{3} = \frac{192}{3} \)  
   \( \left(x^{3/4}\right)^{1/3} = \left(6^4\right)^{1/3} \)  
   \( x = 6^4 \cdot \left(6^4\right)^{1/4} = 4^4 \cdot \sqrt[4]{256} \)  
   \( x = \sqrt[4]{256} \)  
   Check:  
   \( (256)^{3/4} = 192 \)

21. Which gives the solution of \( \sqrt[3]{5x+4} + 5 = 6 ? \)
   a. \( \frac{3}{5} \)  
   b. \( \frac{3}{7} \)  
   c. 1  
   d. none of these  
   \( \frac{3}{5} \)
22. Solve \( (\sqrt{5x - 4})^4 = 2^4 \)\n\[ 5x - 4 = \frac{16}{4} \]
\[ 5x - 4 \geq \frac{20}{5} \]
\[ x \geq 4 \]

23. Solve \( \sqrt{2x + 1} = \sqrt{5 - x} + 2 \). Check for extraneous solutions.

24. For a mammal, the heart rate \( r \) (in beats per minute) and brain weight \( w \) (in kilograms) are related to the body mass \( m \) (in kilograms) by the functions \( r = 241m^{-1/4} \) and \( w = 9.1m^{3/4} \).

a. Solve the function \( r = 241m^{-1/4} \) for \( m \).

b. Solve the function \( w = 9.1m^{3/4} \) for \( m \).

c. Explain how to use the answers from parts (a) and (b) to express the heart rate \( r \) as a function of the brain weight \( w \). Then find the function:

25. It takes a train 8 hours to travel from Capital City to Johnson City when it travels at a speed of 65 mi/h. How long would it take the train to go the same distance when it travels 40 mi/h?

\[ \text{c. } 13 \text{ h} \]

26. The wattage rating \( W \) (in watts) of an appliance varies jointly with the square of the current \( I \) (in amperes) and the resistance \( R \) (in ohms). If the wattage is 6 watts when the current is 0.2 ampere and the resistance is 150 ohms, find the wattage when the current is 0.3 ampere and the resistance is 300 ohms.

\[ \text{a. } 180 \text{ watts} \]
\[ \text{b. } 90 \text{ watts} \]
\[ \text{c. } 27,000 \text{ watts} \]
\[ \text{d. } 27 \text{ watts} \]

27. Write an equation for the relationship:
\[ y \text{ varies directly with the square root of } x \text{ and inversely with } t \]
\[ y = \frac{k \sqrt{x}}{t} \]

28. The intensity \( I \) (in foot-candles) of light received from a source varies inversely with the square of the distance \( d \) from the source. If the light intensity is 5 foot-candles from 16 feet, find the light intensity from 19 feet. Round your answer to the nearest hundredth, if necessary.
\[ I = \frac{k}{d^2} \]
\[ 5 = \frac{k}{16^2} \]
\[ k = 1280 \]
\[ I = \frac{1280}{19^2} \]
\[ I = 32.55 \]

29. Let \( f(x) = \frac{6x + 7}{2x + 1} \). Find the asymptotes of the graph of \( f \), and tell how the graph is related to a hyperbola with equation of the form \( y = \frac{a}{x} \).

\[ \text{VA}: \text{set denominator } = 0 \]
\[ 2x + 1 = 0 \]
\[ x = -\frac{1}{2} \]

\[ \text{HA}: \text{degree in numerator } = \text{degree in denominator} \]
\[ y = \frac{6}{2} = 3 \]
\[ \Rightarrow |y = 3| \]

30. \( f(x) = \frac{4x + 7}{x^2 + 6x + 8} \) \( = \frac{4x + 7}{(x+2)(x+4)} \)

\[ \text{VA}: \text{set denominator } = 0 \]
\[ x = -2, x = -4 \]

\[ \text{a. } x = -4, x = -2, x = -1 \]
\[ \text{b. } \text{none} \]
\[ \text{c. } x = -4, x = -1 \]
\[ \text{d. } x = -4, x = -2 \]
23. \((\sqrt{2x+1})^2 = (\sqrt{5-x+2})^2\)

\[2x+1 = (\sqrt{5-x} + 2)(\sqrt{5-x} + 2)\]
\[2x+1 = 5-x + 2\sqrt{5-x} + 2\sqrt{5-x} + 4\]
\[2x+1 = 9 - x + 4\sqrt{5-x}\]
\[x - 9 = -4 + x\]
\[
\frac{3x - 8}{2} = \frac{1}{4} \sqrt{5-x} \quad \text{Square both sides}
\]

\[
\left(\frac{3x - 8}{2}\right)^2 = \left(\frac{1}{4} \sqrt{5-x}\right)^2
\]
\[
\left(\frac{3x - 8}{2}\right)^2 = 5 - x
\]
\[
\frac{9}{16} x^2 - \frac{3}{2} x - \frac{3}{2} x + 4 = 5 - x
\]
\[
\frac{9}{16} x^2 - 3x + 4 = \frac{6}{5} - x
\]
solve
\[
\frac{9}{16} x^2 + \frac{3}{5} x + \frac{5}{5} + 4 = 0
\]

\[16 \left(\frac{9}{16} x^2 - 2x - 1 = 0\right) \text{ Factor and solve.}
\]
\[9 x^2 - 32x - 16 = 0 \quad \text{ac}=144 \quad \text{-36} \quad \frac{4}{9}
\]
\[9x^2 - 36x + 4x - 16 = 0
\]
\[9x(x-4) + 4(x-4) = 0
\]
\[(9x+4)(x-4) = 0
\]
\[x = -\frac{4}{9} \quad \text{and} \quad x = 4
\]

**Check:**

1. \(x = -\frac{4}{9}\)
   \[\sqrt{2(-\frac{4}{9})+1} = \sqrt{\frac{5}{-\frac{4}{9}}} + 2\]
   \[\sqrt{\frac{5}{9}} + 1 = \frac{5}{\frac{4}{9}} + 2 \quad 5 = \frac{45}{9}
\]
2. \(x = 4\)
   \[\sqrt{2(4)+1} = \sqrt{5-4} + 2 \quad \sqrt{1} = \sqrt{1+2} \quad \frac{3}{2} + 2 = \frac{7}{3} + 2 \quad \text{FALSE!}
\]
31. \( f(x) = \frac{x^2}{x^2 - 4} \)  

\[ y = \frac{x^2}{(x-2)(x+2)} \Rightarrow \text{VA: } x=2, x=-2 \]

Identify all vertical and horizontal asymptote(s) of the graph of the function.

32. \( f(x) = \frac{-x^3}{x^3 - 8} = \frac{-x^3}{(x-2)(x^2+2x+4)} \)

\[ \text{VA: } x=2 \]

\[ \text{HA: } y = \frac{-x^3}{x^3} = -1 \]

33. Use the rational function \( f(x) = \frac{3x^2 - 4x + 9}{5x^2 + 8x - 3} \)

a. What is the horizontal asymptote of \( f(x) \)? \( \text{HA: } y = \frac{3}{5} \)

b. Give an example of another rational function with a quadratic polynomial in the numerator that has the same horizontal asymptote as \( f(x) \).

\[ g(x) = \frac{3x^2 + 7x - 9}{5x^2 + 9x + 1} \]

(need same leading terms)

34. Compare the graphs of \( y = \frac{x^2 + 4}{x - 16} \) and \( y = \frac{x^4 - 16}{x^2 + 4} \). Include x- and y-intercepts, asymptotes, and end behavior in your analysis.

Set \( x = 0 \)

\[ y = \frac{x^2 + 4}{(x-2)(x+2)(x^2+4)} \]

No \( x \)-int.

\( y \)-int.: \((0, \frac{1}{16})\) \( \) \( y \)-int.: \((0, \frac{1}{16})\)

No \( \text{VA} \), No \( \text{HA} \)

35. \( \frac{x^2 - x - 12}{x^2 + x - 20} \)

\[ \text{Divide the expressions. Simplify the result.} \]

36. \( \frac{x^2 - 4x + 4}{15x} \)

\[ \text{Divide the expressions. Simplify the result.} \]

37. \( \frac{x^2 + 9x + 18}{x^2 - 9} + \frac{x + 6}{x - 6} \)

\[ \frac{(x+3)(x+6)}{(x-3)(x+3)} \cdot \frac{(x-6)}{(x+3)(x+6)} = \frac{x^6}{x^3} \]

a. \( \frac{9x + 6}{3} \)

b. \( \frac{x - 6}{x - 3} \)

c. \( \frac{x - 9}{x - 3} \)

d. \( \frac{x + 3}{x - 6} \)

38. \( \frac{x^2 y^3}{3x^4} \cdot \frac{(xy)^2}{x^3 y} \cdot \frac{x^2 y^4}{6y^3} \)

\[ \frac{x^2 y^3}{3x^4} \cdot \frac{x^2 y^2}{x^3 y} \cdot \frac{2y^3}{x^2 y^4} = \frac{2y^3}{x^5} \]
39. Use the rational expression \( \frac{2x^2 - 11x + 12}{x^2 - 7x + 12} \).

a. Factor the expression. Then simplify, if possible.

b. Is the expression in part (a) equal to \( \frac{2x^3 + x^2 - 76x + 105}{(x+5)(x-3)} \)? Explain.

c. Describe the error in simplifying the rational expression:

\[ \frac{\frac{x+2}{x-3}}{x-3} = \frac{x+2}{x-3} \cdot \frac{x-3}{1} = x+2. \]

40. A company is designing packaging in the shape of a right cylinder for a new product. They want the radius of the cylinder to be 2 inches less than its height.

a. Find the volume of the cylinder in terms of the height.

b. Find the surface area of the cylinder in terms of the height.

c. Write the ratio of the volume of the cylinder to the surface area of the cylinder in simplest form.

d. The company decides to choose between a height of 8 inches or 10 inches. They want to choose the one which will be the more cost efficient. Which should they choose? Explain.

Perform the indicated operation(s) and simplify.

41. \[ \frac{-5x}{y^2z^3} + \frac{3}{y^2z^3} = \frac{-5x + 3}{y^2z^2} \]

42. \[ \frac{3x - 5}{x^2 - 25} - \frac{2}{x + 5} = \frac{3x - 5 - 2x + 10}{x^2 - 25} = \frac{x + 5}{x - 5} = \frac{1}{x - 5} \]

Simplify the complex fraction.

\[ \frac{2}{x - 6} \cdot \frac{x - 6}{3 + 5x} = \frac{2}{x - 6} \cdot \frac{3 + 5x}{x} = \frac{2}{x} \cdot \frac{3 + 5x}{x + 6} \cdot \frac{x - 6}{x + 6} = \frac{2x - 27}{5x} \]

Solve the equation. Check for extraneous solutions.

\[ \frac{a}{a - 12} + \frac{4}{a} = 1 \]

a. 1 b. 4 c. 3 d. 2
45. \( \frac{e^5 + 5}{e^5} = \frac{2e + 24}{(e + 5)(e - 5)} \)  
   \( (e + 5)(e - 5) = e^2 - 25 \)  
   \( e^2 - 25 = 2e + 24 \)
   a. -1  
   b. 0  
   c. 1  
   d. 2

46. Which equation shows inverse variation between \( x \) and \( y \)?
   a. \( x - y = 1 \)  
   b. \( xy = 6 \)  
   c. \( y = -3x \)  
   d. \( \frac{y}{x} = 7 \)

47. If \( y \) varies inversely with \( x \), and \( y = 8 \) when \( x = 6 \), what is \( y \) when \( x = 4 \)?
   a. 16  
   b. 6  
   c. 10  
   d. 12

48. The variable \( z \) varies jointly with the variables \( x \) and \( y \). When \( z = 60 \), \( x = 3 \) and \( y = 4 \). Which equation relates \( x \), \( y \), and \( z \)?
   a. \( z = xy + 48 \)  
   b. \( z = 72 - xy \)  
   c. \( z = 5xy \)  
   d. \( z = \frac{80x}{y} \)

49. What are the equations for the asymptotes of the function \( y = \frac{-2}{x + 3} - 5 \)?
   a. \( x = -3, y = -5 \)  
   b. \( x = 3, y = 5 \)  
   c. \( x = 3, y = -5 \)  
   d. \( x = -3, y = 5 \)

50. What are the domain and range of the function \( y = \frac{5}{x - 1} + 2 \)?
   a. domain: \( x \neq 1 \); range: \( y > 2 \)  
   b. domain: \( x = 1 \); range: \( y < 2 \)  
   c. domain: \( x = 1 \); range: \( y > 2 \)  
   d. domain: \( x = 1 \); range: \( y > -2 \)
51. Which graph represents the function \( y = \frac{-1}{x+2} + 4 \)?

- **a.**
- **b.**
- **c.**
- **d.**

52. Simplify the expression \( \frac{3x^2 + 21x + 36}{6x + 24} \).

- **a.** \( x + 3 \)
- **b.** \( \frac{x+3}{2} \)
- **c.** \( \frac{(x-3)(x-4)}{2(x+4)} \)
- **d.** \( \frac{1}{2} \)

53. Multiply: \( \frac{x^2 - 9}{x+3} \cdot \frac{x-5}{x^2 + 2x - 15} \).

- **a.** \( 1 \)
- **b.** \( \frac{(x-3)(x-5)}{(x+3)(x+5)} \)
- **c.** \( \frac{x-3}{x+3} \)
- **d.** \( \frac{x-5}{x+5} \)
54. Divide: \( \frac{9x^2 + 18x - 72}{x^2 - 4x + 4} + \frac{3x^2 + 15x + 12}{x^2 - 3x + 2} \)

\[ \frac{9(x^2+2x-8)}{(x-2)(x-2)} + \frac{(x-2)(x+1)}{3(x^2+5x+4)} \]

\[ \frac{3}{x+1} \]

55. Simplify \( \frac{x^2 - 64}{x + 8} \cdot \frac{x + 8}{x + 3} \)

\( \frac{(x + 8)^2 - (x - 8)}{(x + 3)^2} \)

\( \frac{3(x-1)}{x+1} \)

56. Subtract: \( \frac{2x + 3}{x + 2} - \frac{x + 1}{x + 2} \)

\( \frac{2x + 3 - (x+1)}{x^2} \)

\( \frac{3x + 4}{x + 2} \)

57. Add: \( \frac{5x}{x^2 + 4x + 4} + \frac{7x + 1}{6x + 12} \)

\( \frac{5x}{(x+2)^2} \cdot \frac{7x+1}{6(x+2)} \)

\( \frac{7x^2 + 39x + 2}{6(x+2)(x+2)} \)

58. Solve \( \frac{8}{x+1} = \frac{15}{2x+1} \)

\( \frac{16x + 8}{10x} = \frac{15x - 15}{18x} \)

\( x = 7 \)

59. Solve \( 4 + \frac{6}{x-3} = \frac{2x}{x-3} \)

\( x = 3 \)
60. Solve \( \frac{x}{3} - \frac{5}{x+4} = \frac{-3}{3x+12} \).

\( a. \) \(-6\) or \(2\)
\( b. \) \(6\) or \(-2\)
\( c. \) \(-6\)
\( d. \) \(2\)

\[ 3(x+4) \left[ \frac{x}{3} - \frac{5}{x+4} = \frac{-3}{3(x+4)} \right] \]

\[ x(x+4) - 5(3) = -3 \]
\[ x^2 + 4x - 15 = \frac{-3}{3} \]

\[ x^2 + 4x - 12 = 0 \]
\[ (x+6)(x-2) = 0 \]
\[ x = -6 \] or \( x = 2 \)

Both solutions are valid.

Check:

\( x = -6 \)

\( \frac{-6}{3} - \frac{5}{-6+4} = \frac{-3}{3(-6)+12} \)

\( -2 - \frac{5}{-2} = \frac{-3}{-18+12} \)

\( -2 + \frac{5}{2} = \frac{-3}{-6} \)

\( \frac{1}{2} = \frac{1}{2} \) \( \checkmark \)

\( x = 2 \)

\( \frac{2}{3} - \frac{5}{2+4} = \frac{-3}{3(2)+12} \)

\( \frac{2}{3} - \frac{5}{6} = \frac{-3}{18} \)

\( \frac{2}{3} - \frac{5}{6} = \frac{-1}{6} \)

\( \frac{4}{6} - \frac{5}{6} = \frac{-1}{6} \) \( \checkmark \)