

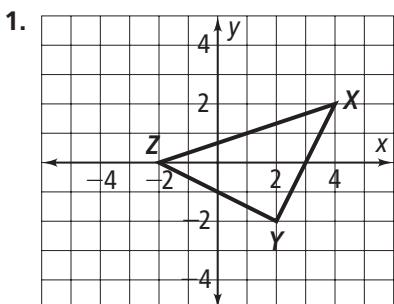
6-7

Practice

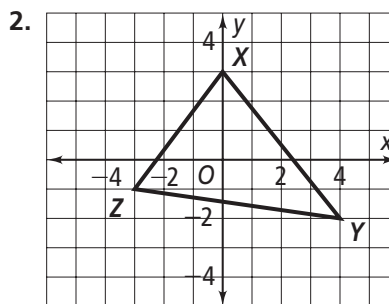
Form G

Polygons in the Coordinate Plane

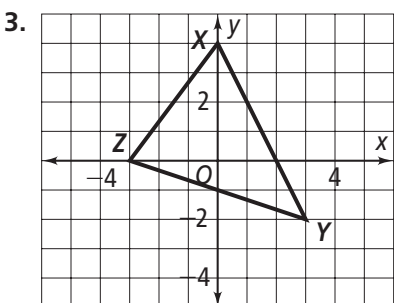
Determine whether $\triangle XYZ$ is *scalene*, *isosceles*, or *equilateral*.



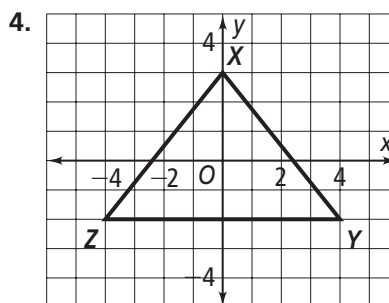
isosceles



scalene

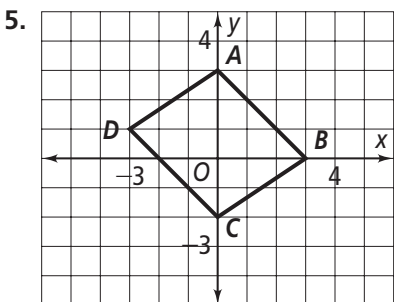


scalene

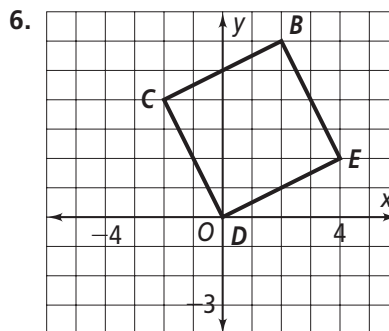


isosceles

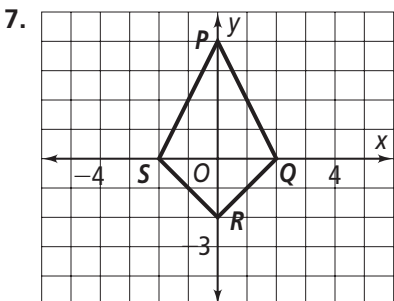
What is the most precise classification of the quadrilateral formed by connecting in order the midpoints of each figure below?



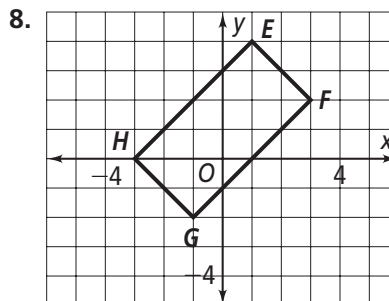
parallelogram



square



rectangle



rhombus

6-7

Practice (continued)

Form G

Polygons in the Coordinate Plane

9. Writing Describe two ways in which you can show whether a parallelogram in the coordinate plane is a rectangle.

Determine whether the diagonals are congruent using the Distance Formula or determine if consecutive sides are perpendicular using the Slope Formula.

10. Writing Describe how you can show whether a quadrilateral in the coordinate plane is a kite.

Determine whether two pairs of consecutive sides are congruent, using the Distance Formulas, and that the pairs are not congruent to each other.

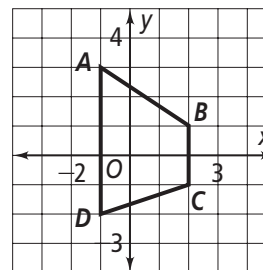
Use the trapezoid at the right for Exercises 11 and 12.

11. Is the trapezoid an isosceles trapezoid? Explain.

No; no sides are congruent.

12. Is the quadrilateral formed by connecting the midpoints of the trapezoid a parallelogram, rhombus, rectangle, or square? Explain.

Parallelogram; the slopes of the opposite sides are equal, but adjacent sides are not perpendicular and the sides are not all congruent.



Determine the most precise name for each quadrilateral. Then find its area.

13. $A(-6, 3), B(-2, 0), C(-2, -5), D(-6, -2)$ **rhombus; 20 units²**

14. $A(1, 8), B(4, 6), C(1, -2), D(-2, 0)$ **parallelogram; 30 units²**

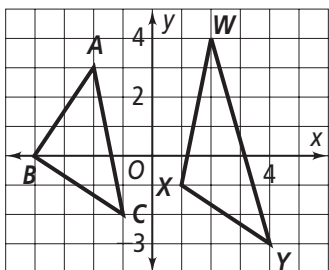
15. $A(3, 4), B(8, 1), C(2, -9), D(-3, -6)$ **rectangle; 68 units²**

16. $A(0, -1), B(1, 4), C(4, 3), D(3, -2)$ **parallelogram; 16 units²**

17. $A(-5, 14), B(-2, 11), C(-5, 8), D(-8, 11)$ **square; 18 units²**

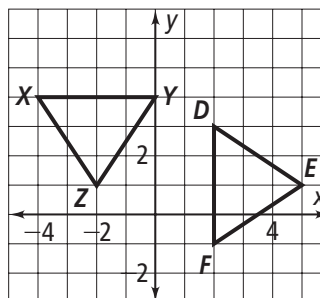
Determine whether the triangles are congruent. Explain.

18.



No; corresponding sides not \cong .

19.



Yes; corr. sides are \cong .

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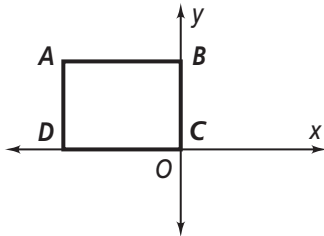
Practice

Form G

Applying Coordinate Geometry

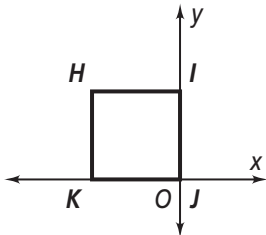
Algebra What are the coordinates of the vertices of each figure?

1. rectangle with base b and height h



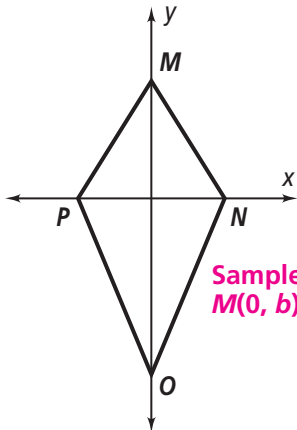
$A(-b, h); B(0, h); C(0, 0); D(-b, 0)$

3. square with height x



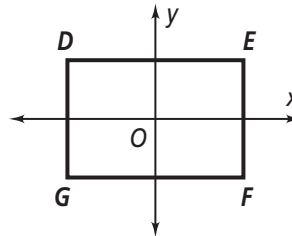
$H(-x, x); I(0, x); J(0, 0); K(-x, 0)$

5. kite $MNOP$ where $PN = 4s$ and the y -axis bisects \overline{PN}



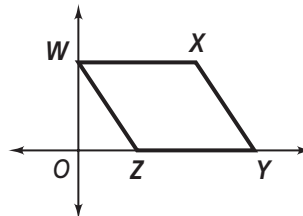
Sample: $M(0, b); N(2s, 0); O(0, -c); P(-2s, 0)$

2. rectangle centered at the origin with base $2b$ and height $2h$



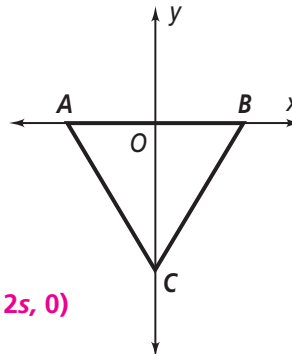
$E(b, h); F(b, -h); G(-b, -h); D(-b, h)$

4. parallelogram with height m and point Z distance j from the origin



Sample: $W(0, m); X(x - j, m); Y(x, 0); Z(j, 0)$

6. isosceles $\triangle ABC$ where $AB = 2n$ and the y -axis is the median



Sample: $A(-n, 0); B(n, 0); C(0, -y)$

7. How can you determine if a triangle on a coordinate grid is an isosceles triangle? **Use the Distance Formula to compare side lengths.**
8. How can you determine if a parallelogram on a coordinate grid is a rhombus? **Answers may vary. Sample: Use the Slope Formula to determine if the diagonals are perpendicular.**
9. How can you determine if a parallelogram on a coordinate grid is a rectangle? **Answers may vary. Sample: Use the Distance Formula to determine if the diagonals are congruent.**

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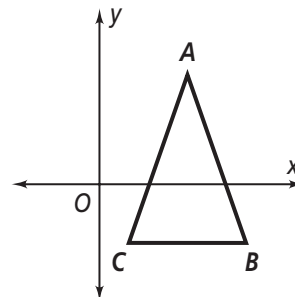
Practice (continued)

Form G

Applying Coordinate Geometry

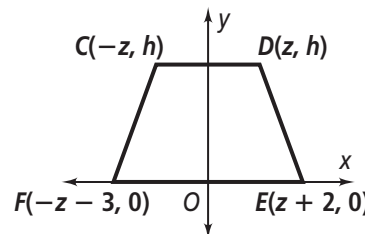
10. In the triangle at the right, A is at $(m + r, s)$, B is at $(2m, -p)$, and C is at $(2r, -p)$. Is this an isosceles triangle? Explain.

Yes; $AC = AB$. Both are equal to $\sqrt{m^2 - 2rm + r^2 + s^2 + 2sp + p^2}$.



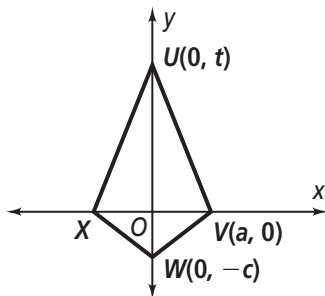
11. Is the trapezoid shown at the right an isosceles trapezoid? Explain.

No; the diagonals are not congruent. $CE = \sqrt{(2z + 2)^2 + h^2}$, $DF = \sqrt{(2z + 3)^2 + h^2}$

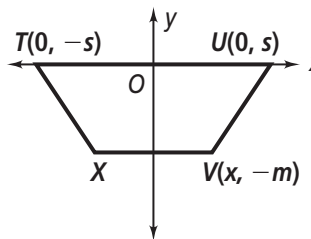


For Exercises 12 and 13, give the coordinates for point X without using any new variables.

12. Kite $(-a, 0)$

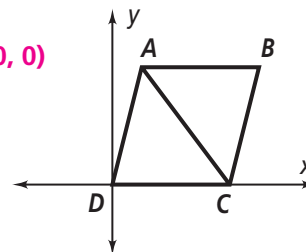


13. $TX = UV$ $(-x, -m)$



14. Plan a coordinate proof to show that either diagonal of a parallelogram divides the parallelogram into two congruent triangles.

- Name the coordinates of parallelogram $ABCD$ at the right.
Answers may vary. Sample: $A(p, r)$; $B(p + m, r)$; $C(m, 0)$; $D(0, 0)$
- What do you need to do to show that $\triangle ACD$ and $\triangle CAB$ are congruent?
Show that corresponding sides are \cong .
- How will you determine that those parts are congruent?
Use the Distance Formula.



Classify each quadrilateral as precisely as possible.

- $A(-3a, 3a)$, $B(3a, 3a)$, $C(3a, -3a)$, $D(-3a, -3a)$ **square**
- $A(c, d + e)$, $B(2c, d)$, $C(c, d - 2e)$, $D(0, d)$ **kite**

6-9

Practice

Form G

Proofs Using Coordinate Geometry

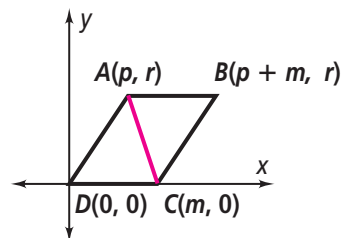
Use coordinate geometry to prove each statement. Follow the outlined steps.

1. Either diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Given: $\square ABCD$

Prove: $\triangle ACD \cong \triangle CAB$

- a. Use the figure at the right. Draw \overline{AC} .



- b. Which theorem should you use to show that $\triangle ACD$ and $\triangle CAB$ are congruent? Explain.

SSS; side lengths can be shown to be equal using coordinate geometry.

- c. Which formula(s) will you need to use?

Distance Formula

- d. Show that $\triangle ACD$ and $\triangle CAB$ are congruent.

$AD = CB$. By the Distance Formula, both are equal to $\sqrt{p^2 + r^2}$. $AB = CD$. By the Distance Formula, both are equal to m . $AC = CA$ by the Reflexive Property of Equality. So, the triangles are congruent by SSS.

2. The diagonals of a parallelogram bisect one another.

Given: $\square ABCD$

Prove: The midpoints of the diagonals are the same.

- a. How will you place the parallelogram in the coordinate plane?

Answers may vary. Sample: with vertices $A(p, r)$; $B(p + m, r)$; $C(m, 0)$; and $D(0, 0)$

- b. Find the midpoints of \overline{AC} and \overline{BD} . What are the coordinates

of the midpoints? **Answers may vary. Sample: The midpoint of AC is $(\frac{p+m}{2}, \frac{r}{2})$; the midpoint of BD is $(\frac{p+m}{2}, \frac{r}{2})$.**

- c. Are the midpoints the same? Do the diagonals bisect one another?

yes; yes

- d. **Reasoning** Would using a different parallelogram or labeling the vertices differently change your answer? Explain.

Answers may vary. Sample: No; the midpoints would still have identical coordinates.

3. How can you use coordinate geometry to prove that if the midpoints of a square are joined to form a quadrilateral, then the quadrilateral is a square? Explain.

Use the Midpoint Formula to find the midpoints of the square. Then use the Slope Formula to show that consecutive sides are perpendicular, and the Distance Formula to show that all sides are congruent.

6-9

Practice (continued)

Form G

Proofs Using Coordinate Geometry

Tell whether you can reach each conclusion below using coordinate methods.

Give a reason for each answer.

4. A triangle is isosceles.

Yes; use the Distance Formula. You would need to prove that two sides of the triangle are congruent. You could do this by finding the distances between the points that form the triangle.

5. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

Yes; find the midpoint of the hypotenuse by using the Midpoint Formula. Then find the distance of this midpoint from each vertex by using the Distance Formula.

6. If the midpoints of the sides of an isosceles trapezoid are connected, they will form a parallelogram.

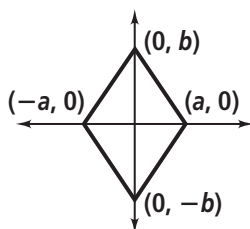
Yes; find the midpoints of the sides by using the Midpoint Formula. Then use the Slope Formula to find the slopes of the segments formed by connecting these midpoints. If opposite sides are parallel, then the figure formed is a parallelogram.

7. The diagonals of a rhombus bisect one another.

Yes; use the Midpoint Formula to find the midpoint of the diagonals. If the midpoints are the same, then the diagonals bisect one another.

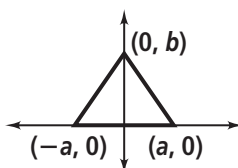
Use coordinate geometry to prove each statement.

8. The segments joining the midpoints of a rhombus form a rectangle.



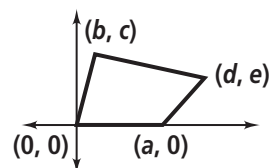
The midpoints are $(\frac{a}{2}, \frac{b}{2})$, $(-\frac{a}{2}, \frac{b}{2})$, $(-\frac{a}{2}, -\frac{b}{2})$, and $(\frac{a}{2}, -\frac{b}{2})$. The quadrilateral formed by these points has sides with vertical and horizontal slopes. Therefore, the consecutive sides are perpendicular, making the quadrilateral a rectangle.

9. The median to the base of an isosceles triangle is perpendicular to the base.



The median meets the base at $(0, 0)$, the midpoint of the base. Therefore, the median has undefined slope, or is vertical. Because the base is a horizontal segment, the median is perpendicular to the base.

10. The segments joining the midpoints of a quadrilateral form a parallelogram.



The midpoints are $(\frac{a}{2}, 0)$, $(\frac{a+d}{2}, \frac{e}{2})$, $(\frac{b+d}{2}, \frac{c+e}{2})$, and $(\frac{b}{2}, \frac{c}{2})$. One pair of opposite sides has a slope of $\frac{e}{d}$, and the other pair has a slope of $\frac{c}{b-a}$. Therefore, it is a parallelogram.