17.4 Series and Parallel Circuits

When multiple resistors are used in a circuit, the total resistance in the circuit must be found before finding the current. Resistors can be combined in a circuit in series or in parallel.

Resistors in Series

When connected in series, the total resistance, R_T, is equal to

$$R_T = R_1 + R_2 + R_3 + \ldots$$

In series, the total resistance is always *larger* than any individual resistance.

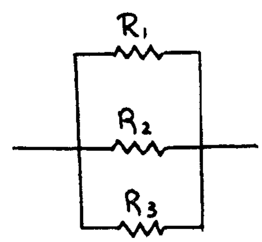
Current in series resistors: In series circuits, charge has only one path through which to flow. Therefore, the current passing through each resistor in series is the same.

Potential difference across series resistors: As charge passes through each of the resistors, it loses some energy. This means that there will be a potential difference across each resistor. The sum of all the potential differences equals the potential difference across the battery, assuming negligible resistance in the connecting wires.

Resistors in Parallel

When connected in parallel, the total resistance, R_T, is equal to

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



Don't forget! After finding a common denominator and determining the sum of these fractions, flip over the answer to determine R_T.

In parallel circuits, the total resistance is always *smaller* than any individual resistance.

Current in parallel resistors: In parallel circuits, there is more than one possible path and current divides itself according to the resistance of each path. Since current will take the "path of least resistance," the smallest resistor will allow the most current through, while the largest resistor will allow the least current through. The sum of the currents in each parallel resistor equals the original current entering the branches.

Potential difference in parallel resistors: The potential difference across each of the resistors in a parallel combination is the same. If there are no other resistors in the circuit, it is equal to the potential difference across the battery, assuming negligible resistance in the connecting wires.

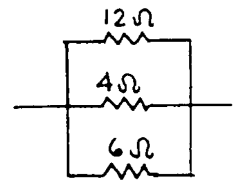
Example 8: Find the total resistance of the three resistors connected in series.

Solve: $R_T = R_1 + R_2 + R_3 = 12\ \Omega + 4\ \Omega + 6\ \Omega = \mathbf{22\ \Omega}$

Example 9: Find the total resistance of the same three resistors now connected in parallel.

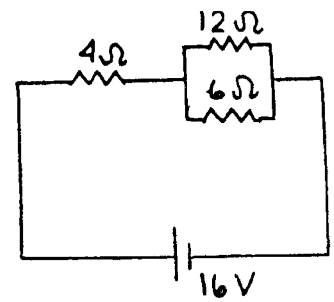
Solve: $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega}$$

$$\frac{1}{R_T} = \frac{1}{12\ \Omega} + \frac{3}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{6}{12\ \Omega} = \frac{1}{2\ \Omega} \qquad R_T = \mathbf{2\ \Omega}$$



Example 10: Find the total current passing through the circuit.

This circuit contains resistors in parallel that are then combined with a resistor in series. Always begin solving such a resistor combination by working from the inside out. In other words, first determine the equivalent resistance of the two resistors in parallel before combining this total resistance with the one in series.



Look first at the parallel combination.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} = \frac{1}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{3}{12\ \Omega} = \frac{1}{4\ \Omega}$$

$$R_T = 4\ \Omega$$

Now, combine this equivalent resistance with the resistor in series.

$$R_T = R_1 + R_2 = 4\ \Omega + 4\ \Omega = 8\ \Omega$$

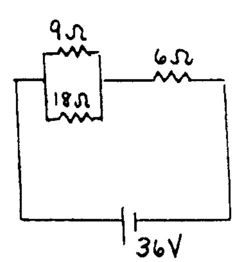
To find the current flowing through the circuit, use this total resistance in combination with the potential difference from the battery.

Given: $V = 16\ \text{V}$
$R = 8\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{16\ \text{V}}{8\ \Omega} = \mathbf{2\ A}$

Example 11: Find the current in the 9-Ω resistor.



For the parallel branch

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{9\ \Omega} + \frac{1}{18\ \Omega} = \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega} = \frac{3}{18\ \Omega} = \frac{1}{6\ \Omega}$$

$$R_T = 6\ \Omega$$

Combining with the series resistor

$$R_T = R_1 + R_2 = 6\ \Omega + 6\ \Omega = 12\ \Omega$$

Given: $V = 36\text{ V}$
$R = 12\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{36\text{ V}}{12\ \Omega} = 3\text{ A}$

This 3 A is the current through the entire circuit. Use this current to find the potential difference across the parallel combination. Remember, the potential difference across resistors wired in parallel is the same regardless of which path is taken. Because the resistors in parallel have a combined resistance of 6 Ω, you find the potential difference across the parallel branch as follows.

Given: $R = 6\ \Omega$
$I = 3\text{ A}$

Unknown: $V = ?$
Original equation: $V = IR$

Solve: $V = IR = (3\text{ A})(6\ \Omega) = 18\text{ V}$

Therefore, the potential difference across both the top and the bottom branches is 18 V. Now use this 18-V drop to determine the current in the 9-Ω resistor.

Given: $V = 18\text{ V}$
$R = 9\ \Omega$

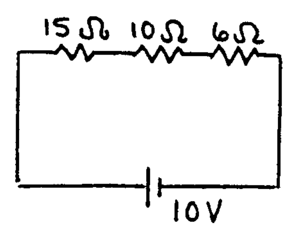
Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{18\text{ V}}{9\ \Omega} = \mathbf{2\text{ A}}$

Practice Exercises

Exercise 16: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.

a) $R_T = R_1 + R_2 + R_3 = 15\ \Omega + 10\ \Omega + 6\ \Omega = \mathbf{31\ \Omega}$
b) $I = V/R\ (10.0\text{ V})/(31\ \Omega) = \mathbf{0.32\text{ A}}$

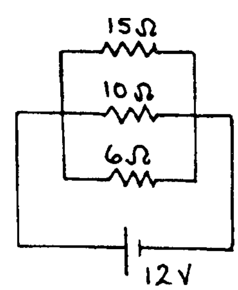


Answer: **a.** **31 Ω**

Answer: **b.** **0.32 A**

Exercise 17: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.

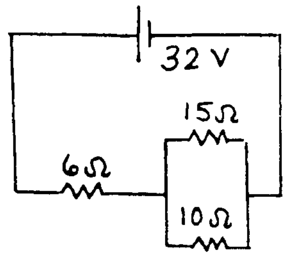
a) $1/R_T = 1/R_1 + 1/R_2 + 1/R_3$
$= 1/15\ \Omega + 1/10\ \Omega + 1/6\ \Omega = 10/30\ \Omega$ $\quad R_T = \mathbf{3\ \Omega}$
b) $I = V/R = (12\text{ V})/(3\ \Omega) = \mathbf{4\text{ A}}$



Answer: **a.** **3 Ω**

Answer: **b.** **4 A**

Exercise 18: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



a) $1/R_T = 1/R_1 + 1R_2 = 1/15\ \Omega + 1/10\ \Omega = 5/30\ \Omega$
$R_T = 6\ \Omega$
$R_T = R_1 + R_2 = 6\ \Omega + 6\ \Omega =$ **12 Ω**
b) $I = V/R = (32\ V)/(12\ \Omega) =$ **2.7 A**

Answer: **a.** **12 Ω**

Answer: **b.** **2.7 A**

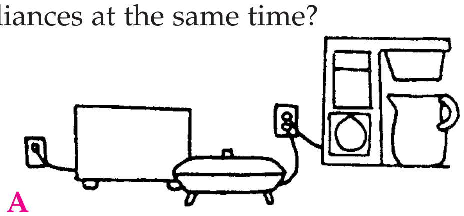
Exercise 19: Old-fashioned holiday lights were connected in series across a 120-V household line. a) If a string of these lights consists of 12 bulbs, what is the potential difference across each bulb? b) If the bulbs were connected in parallel, what would be the potential difference across each bulb?

Series: (120 V)/(12) = **10. V per bulb** Parallel: **120 V per bulb**

Answer: **a.** **10. V per bulb**

Answer: **b.** **120 V per bulb**

Exercise 20: Before going to work each morning, Gene runs his 18-Ω toaster, 11-Ω electric frying pan, and 14-Ω electric coffee maker, all at the same time. The three are connected in parallel across a 120-V line. a) What is the current through each appliance? b) If a household circuit could carry a maximum current of 15 A, would Gene be able to run all of these appliances at the same time?



a) Toaster: $I = V/R = (120\ V)/(18\ \Omega) =$ **6.7 A**
Frying Pan: $I = V/R = (120\ V)/(11\ \Omega) =$ **10.9 A**
Coffee Maker: $I = V/R = (120\ V)/(14\ \Omega) =$ **8.6 A**
b) $I_T = I_1 + I_2 + I_3 = 6.7\ A + 10.9\ A + 8.6\ A = 26.2\ A$
All three can't run at the same time.

Answer: **a.** **6.7 A; 10.9 A; 8.6 A**

Answer: **b.** **see above**

Exercise 21: Timmy is playing with a new electronics kit he has received for his birthday. He takes out four resistors with resistances of 15 Ω, 20 Ω, 20 Ω, and 30 Ω. a) How would Timmy have to wire the resistors so that they would allow the maximum amount of current to be drawn? Calculate the total resistance in this circuit. b) How must he wire the resistors so that they draw a minimum amount of current? Calculate the total resistance in this circuit.

a) Parallel: $1/R_T = 1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 =$
$1/15\ \Omega + 1/20\ \Omega + 1/20\ \Omega + 1/30\ \Omega = 12/60\ \Omega$ $\quad R_T = \mathbf{5\ \Omega}$
b) Series: $R_T = R_1 + R_2 + R_3 + R_4$ $\quad R_T = 15\ \Omega + 20\ \Omega + 20\ \Omega + 30\ \Omega = \mathbf{85\ \Omega}$

Answer: **a.** **5 Ω**

Answer: **b.** **85 Ω**

Exercise 22: Farmer Crockett is preparing tomato seedlings for his spring planting by growing the small plants over five 46-Ω strip heaters wired in parallel. a) How much current does each heater draw from a 120-V line? b) How much current do they draw all together?

a) $I = V/R = (120\ \text{V})/(46\ \Omega) = \mathbf{2.6\ A}$
b) $1/R_T = 5(1/R) = 5/46\ \Omega$ $\quad R_T = 9.2\ \Omega$
$I = V/R = (120\ \text{V})/(9.2\ \Omega) = \mathbf{13\ A}$

Answer: **a.** **2.6 A**

Answer: **b.** **13 A**

Additional Exercises

A-1: Otto accidently leaves his automobile headlights on overnight and is unable to start his car in the morning. Each of the two headlights connected in parallel draws 2.00 A of current from the 12.0-V battery. a) If the battery stores 7.50×10^5 J of energy, how long will it take for the headlights to go off? b) Why are the headlights connected in parallel?

A-2: Officer Moynihan is patrolling his beat with a 4.5-V flashlight whose lightbulb has a resistance of 12 Ω. How much current does the flashlight draw?

A-3: Each night before falling asleep, Linus turns on his electric blanket that is plugged into the 120.-V electrical outlet. A current of 1.20 A flows through the blanket. a) What is the blanket's resistance? b) Does Linus want his electric blanket to have a high resistance or a low resistance? Why?

A-4: Herbert had just suffered a heart attack but he was revived in the hospital emergency room with a device called a defibrillator. (The paddles of a defibrillator supply a short pulse of high voltage to restart the heart.) The defibrillator contains a 20.-μF capacitor that releases 0.15 C of charge. a) What is the potential difference between the defribrillator paddles during the discharge? b) Why do you think doctors yell "Clear!" to the attendants before discharging the defibrillator?

A-5: Sherm is typing his term paper on a computer that contains a high-speed switch, controlled with a small 100×10^{-12} F speed-up capacitor. What is the current flow created by the capacitor if it discharges every 0.1 s across a potential difference of 5 V?

A-6: Every Sunday morning Stuart makes "breakfast in bed" for his wife. However, because the household wires can only carry a maximum current of 15 A from the 120.-V line, it is difficult to run all of the appliances simultaneously without blowing a fuse. What is the most power Stuart may use while cooking, before blowing a fuse?

A-7: In the previous exercise, a) how much current will Stuart draw if he tries to run the 700.-W toaster and 1000.-W coffee maker at the same time? b) Will this cause him to blow the fuse?

A-8: Xiaoyi's aquarium operates for 24.0 h a day and contains a 5.0-W heater, two 20.0-W lightbulbs, and a 35.0-W electric filter. If Xiaoyi pays $0.100 per kWh for her electricity bill, how much will it cost to maintain the aquarium for 30.0 days?

A-9: The average power plant, running at full capacity, puts out 500. MW of power. If the power company charges its customers $0.10 per kWh, what is the revenue brought in by the power plant each day?

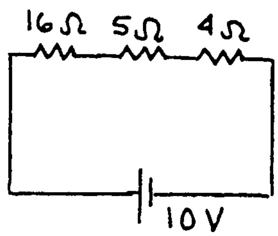
A-10: Horace has invented a unique pair of reading glasses that have two small light bulbs at the bottom wired in series, so that he can see the newspaper when he is reading at night. Each of the bulbs has a resistance of 2.00 Ω, and the system runs off a 3.20-V battery. How much current is drawn by Horace's reading glasses?

A-11: Jay has two 8-Ω stereo speakers wired in series in the front of his car connected to the 4.0-V output of the stereo. a) What is the current through each of the speakers? b) In his garage, Jay finds two more old speakers with resistances of 4 Ω and 16 Ω. He wires each in parallel with the 8-Ω combination. What is the new current through the 8-Ω speakers? c) If the loudness of each speaker is proportional to the amount of power used, how has the loudness of the two 8-Ω speakers changed?

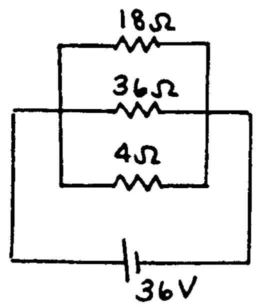
A-12: Find a) the total resistance in circuit A below. b) Find the total current through the circuit.

A-13: Find a) the total resistance in circuit B below. b) Find the total current through the circuit.

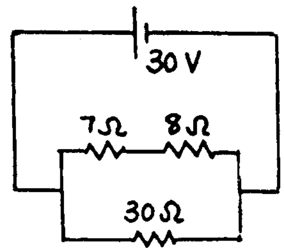
A-14: Find a) the total resistance in circuit C below. b) Find the total current through the circuit.



Circuit A



Circuit B

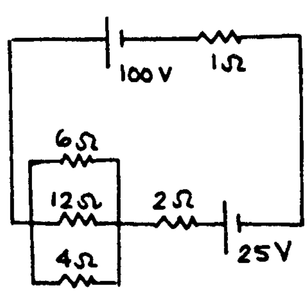


Circuit C

Challenge Exercises for Further Study

B-1: An 800.-W submersible electric heater is put into a 20.0 °C hottub until the 50.0-kg of tub water has warmed up to 70.0 °C. How long will it take for the heater to heat the tub water? (c_{water} = 4187 J/kg°C)

B-2: Find the total current in the circuit in the diagram.



B-3: In exercise A-10, the light bulbs are rated for 5 h of use before they burn out. If the battery can supply 5184 J to the circuit, which occurs first, energy depletion in the battery or failure of a bulb?