

Verona Public School District Curriculum Overview

Algebra 2



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Verona Public Schools
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Verona Public Schools Mission Statement:

The mission of the Verona Public Schools, the center of an engaged and supportive community, is to empower students to achieve their potential as active learners and productive citizens through rigorous curricula and meaningful, enriching experiences.

Course Description:

This course is designed to enhance the concepts developed in Algebra I College Prep A and Geometry College Prep A. Students will continue to improve their ability to model situations and solve a variety of equations including linear, quadratic, rational and radical. Topics for this course include: Polynomial, Rational and Radical Relationships, Trigonometric Functions, and Conics. Graphing calculators will be an important tool that will routinely be used in instruction.

Prerequisite(s): Geometry College Prep.



Standard 8: Technology Standards

8.1: Educational Technology: <i>All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.</i>	8.2: Technology Education, Engineering, Design, and Computational Thinking - Programming: <i>All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.</i>
X A. Technology Operations and Concepts X B. Creativity and Innovation C. Communication and Collaboration D. Digital Citizenship E. Research and Information Fluency X F. Critical thinking, problem solving, and decision making	X A. The Nature of Technology: Creativity and Innovation B. Technology and Society C. Design D. Abilities for a Technological World E. Computational Thinking: Programming

SEL Competencies and Career Ready Practices

Social and Emotional Learning Core Competencies: <i>These competencies are identified as five interrelated sets of cognitive, affective, and behavioral capabilities</i>	Career Ready Practices: <i>These practices outline the skills that all individuals need to have to truly be adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.</i>
Self-awareness: The ability to accurately recognize one's emotions and thoughts and their influence on behavior. This includes accurately assessing one's strengths and limitations and possessing a well-grounded sense of confidence and optimism.	X CRP2. Apply appropriate academic and technical skills. CRP9. Model integrity, ethical leadership, and effective management. CRP10. Plan education and career paths aligned to personal goals.
Self-management: The ability to regulate one's emotions, thoughts, and behaviors effectively in different situations. This includes managing stress, controlling impulses, motivating oneself, and setting and working toward achieving personal and academic goals.	CRP3. Attend to personal health and financial well-being. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP11. Use technology to enhance productivity.
Social awareness: The ability to take the perspective of and empathize with others from diverse backgrounds and cultures, to understand social and ethical norms for behavior, and to recognize family, school, and community resources and supports.	X CRP1. Act as a responsible and contributing citizen and employee. CRP9. Model integrity, ethical leadership, and effective management.
Relationship skills: The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. This includes communicating clearly, listening actively, cooperating, resisting inappropriate social pressure, negotiating conflict constructively, and seeking and offering help when needed.	X CRP4. Communicate clearly and effectively and with reason. CRP9. Model integrity, ethical leadership, and effective management. CRP12. Work productively in teams while using cultural global competence.
Responsible decision making: The ability to make constructive and respectful choices about personal behavior and social interactions based on consideration of ethical standards, safety concerns, social norms, the realistic evaluation of consequences of various actions, and the well-being of self and others.	CRP5. Consider the environmental, social, and economic impact of decisions. CRP7. Employ valid and reliable research strategies. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership, and effective management.

Standard 9: 21st Century Life and Careers

9.1: Personal Financial Literacy: <i>This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.</i>	9.2: Career Awareness, Exploration & Preparation: <i>This standard outlines the importance of being knowledgeable about one's interests and talents, and being well informed about postsecondary and career options, career planning, and career requirements.</i>	9.3: Career and Technical Education: <i>This standard outlines what students should know and be able to do upon completion of a CTE Program of Study.</i>
X A. Income and Careers B. Money Management C. Credit and Debt Management D. Planning, Saving, and Investing X E. Becoming a Critical Consumer F. Civic Financial Responsibility G. Insuring and Protecting	A. Career Awareness (K-4) B. Career Exploration (5-8) X C. Career Preparation (9-12)	A. Agriculture, Food & Natural Res. B. Architecture & Construction C. Arts, AV Technology & Comm. D. Business Management & Admin. E. Education & Training F. Finance G. Government & Public Admin. H. Health Science I. Hospital & Tourism J. Human Services K. Information Technology L. Law, Public, Safety, Corrections & Security M. Manufacturing N. Marketing X O. Science, Technology, Engineering & Math P. Transportation, Distribution & Log.

Course Materials

Core Instructional Materials: <i>These are the board adopted and approved materials to support the curriculum, instruction, and assessment of this course.</i>	Differentiated Resources: <i>These are teacher and department found materials, and also approved support materials that facilitate differentiation of curriculum, instruction, and assessment of this course.</i>
<ul style="list-style-type: none"> Pearson Algebra 2 Common Core © 2011 	<ul style="list-style-type: none"> NJ Model Curriculum Delta Math NJCTL Pearson Sample PARCC items Eureka Math: A Story of Functions Modules Various online sources



Unit Title: Polynomial Equations

Time Frame/Duration: 1 month

Cross-curricular Opportunities: Physics

Stage 1: Desired Results

Established Goals:

A.REI.11 – Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately using technology to graph the functions, make tables of values to find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions

F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts, intervals where the function is increasing, decreasing positive or negative, relative minimum and maximums, symmetries, end behavior and periodicity.

F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^x$, $y = (0.97)^x$, $y = (1.01)^{12}$, $y = (1.2)^{1/10}$, and classify them as representing exponential growth or decay.

F.IF.9 – Compare properties to two functions each function each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions). For examples, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^4$ as $(x^2)^2 - (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.APR.2 – Know and apply the remainder theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.3 – Identify zeros of polynomials when suitable factors are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.

F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.BF.3 – Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.1 – Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Transfer Goal:

Students will be able to independently use their learning to... analyze characteristics of different forms of equations and justify solutions as a basis for further math topics and how they relate to the world around us.

Students will understand that:

- U1. Most problems can be solved in different ways
- U2. Word problems and graphs are tools that can help us make meaningful connections to solve real-world problems.
- U3. Equations and operations can help us make sense of various situations.
- U4. Mathematical Properties can be seen in many other subjects.
- U5. Nonlinear functions make good models of many real life situations.

Students will keep considering:

- Q1. How do you decide which method to use to best solve a problem?
- Q2. What is the relationship between the graph and the equation of a polynomial?
- Q3. How can you determine and use the relationships between equations to further investigate future situations?

Students will know:

- K1. How to interpret imaginary numbers
- K2. How to transform all types of polynomial functions on the coordinate plane
- K3. How to solve various polynomial equations multiple ways (graphing, factoring, formulas, processes)
- K4. How to simplify polynomial expressions through addition, subtraction, and multiplication
- K5. How to determine the number of solutions of a polynomial equation
- K6. How to transform an expression in different forms
- K7. How to divide polynomials
- K8. Key features of polynomial graphs
- K9. How to solve a real life modeling problems
- K10. How to solve systems of equations using various methods (graphing, tables, substitution, elimination) and the meaning of their answers
- K11. How to compute average rates of change

Students will be skilled at:

- A1. Performing computations with polynomials and imaginary numbers
- A2. Factoring polynomials completely
- A3. Rewriting and solve polynomials and different forms
- A4. Interpreting graphs of polynomials
- A5. Analyzing functions and switching between graphical and equation based meanings
- A6. Deciding which method of division is to be used and which is most efficient
- A7. Choosing the best way to solve problems
- A8. Analyzing functions based on average rates of change over specific intervals
- A9. Creating their own problems that represent relationships involving polynomials equations

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

[Polynomials Unit Test Bank](#)

[On Target Project](#) - Students will use quadratic equations in a real-life situations that each can picture and test. Students use tables and graphs to study the paths of arrows and then use their knowledge of the relationships between the graphs and tables to analyze and create equations that model the paths.

[Polynomials/On Target Project Rubric.docx](#)

[Polynomials/On Target Answers.pdf](#)

[Volume Maximization](#) – Students will analyze a cylindrical container in order to maximize its contents while also thinking about why a manufacturer would want to use maximization in their creation of containers. Students will complete calculations along with written questions.

[Polynomials/Volume Maximization - performance task answers.pdf](#)

[Curves by Design](#) – students will adjust a polynomial equation to fit their design of the hood section of a car while also writing equations of cubics and revisiting matrices and regressions.

[Polynomials/Curves By Design - Project Rubric.docx](#)

Lesson Quizzes

[5-1 LQ](#)

[5-2 LQ](#)

[5-3 LQ](#)

[5-4 LQ](#)

[5-5 LQ](#)

[5-6 LQ](#)

[5-7 LQ](#)

[5-8 LQ](#)

[5-9 LQ](#)

Mid-Unit Test (Only for 15-16 school year)

[Polynomials/Algebra 2 Honors - Unit 1 Test 1 - Quadratics.pdf](#)

[Polynomials/Algebra 2 Honors - Unit 1 Test 1 - Quadratics blueprint.xlsx](#)

Final Unit Test

[Polynomials/Algebra 2 Honors - Unit 1 Test 2 - Polynomials.pdf](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal:

- Identify parts of a polynomial while working towards other parts of problems
- Students should also troubleshoot examples and explain misapplications of the conventions or properties.

Reference Materials

[Polynomials Topical Outline.docx](#)

[Next Generation PARCC Resources](#)

[Kahn Academy - Polynomials & Rationals](#)

[Math is Fun - Polynomials](#)

[Spark Note - Polynomials \(Ignore Nested Form\)](#)

[PARCC EOY Assessment Evidence Table](#)

[Click on the specific Standard to find Sample Tasks](#)

[Regents Exam Questions by topic](#)

[Math Videos By Topic](#)

[Math Assessment Project - Tasks by Standard](#)



Unit Title: Rational Equations
Time Frame/Duration: 1 month

Cross-curricular Opportunities: Physics, Chemistry, Manufacturing, Business Modeling

Stage 1: Desired Results

Established Goals

F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.^{QUA}

Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.BF.1 – Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F.BF.3 – Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A.SSE.1 – Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .

A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$

A.REI.1 – Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.2 – Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.11 – Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately using technology to graph the functions, make tables of values to find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions.

Transfer Goal:

Students will be able to independently use their learning to...further understand the different function families and their properties while relating the operations performed on these functions to previous topics

Students will understand that:

- U1. Using ideas and properties about simple fractions can make rational expressions much easier.
- U2. Ratios and proportions are essential in everyday life especially, but not limited to, percentages and discounts.
- U3. You can add, subtract, multiply and divide functions just like regular numbers, but just make sure the denominator doesn't equal 0.
- U4. The degrees of the numerator and denominator determine the shape of a graph.
- U5. Domain becomes increasingly more important in more complex function families.

Students will keep considering:

- Q1. Why must we follow rules and properties in Algebra?
- Q2. In what situations are some answers to problems not correct?
- Q3. How are fractions helpful?
- Q4. How are properties of rational functions similar to properties of polynomials?
- Q5. How can you tell what numbers are allowed to be substituted for x ?

Students will know:

- K1. How to perform operations on rational expressions and determine the domain
- K2. How to translate Inverse function family
- K3. How to interpret different parts of functions and how these part affect the graph
- K4. How to write rational expressions in different forms (performing operations and making common denominators)
- K5. How to graph and find all key features of graphs
- K6. How to solve rational equations

Students will be skilled at:

- A1. Solving problems involving rates with variables
- A2. Interpreting solutions algebraically and graphically
- A3. Creating and solve rational equations that represent situations
- A4. Understanding extraneous solutions and why they exist
- A5. Making comparisons between rational functions and other functions they have learned about in the past.
- A6. Using the graphing calculator to make graphing and analysis quicker and easier

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- Rational Expressions Test Bank
- [Under Pressure Project](#) - Students will use ideas about rational equations to explore safety issues relating to scuba diving.
 - [Under Pressure Project Rubric](#)
- [Rational Expressions Task](#) - Student will use ideas about rational equations and formulas to answer a series of questions on designing a beverage container
 - [Task Rubric](#)
 - [Task Answers](#)
- [Rational Expression Transfer Task](#) - Students will form their own real-life examples of rational expressions requiring them to think about the different parts of graphs and equations that would relate to a situation encountered in the world around them.
- Lesson Quizzes
 - [8-1 LQ](#)
 - [8-2 LQ](#)
 - [8-3 LQ](#)
 - [8-4 LQ](#)
 - [8-5 LQ](#)
 - [8-6 LQ](#)
- Mid Unit Quiz
 - [Mid Unit Quiz](#)
- Unit Test
 - [Unit 2 Test Review](#)
 - [Unit 2 Test Review Answers](#)
 - [Unit 2 Test](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

- Formal:
- Providing written/oral response to the EQs
 - Passing all quizzes on basic concepts in unit.
- Informal:
- Identifying all aspects of a rational function's graph, comparing and contrasting polynomial functions
 - Troubleshoot examples explaining errors in reasoning and misuse of conventions or properties
 - Being able to explain why concepts regarding fractions are essential to master BEFORE fully understanding Rational expressions

Reference Materials

- [Rational Expressions Topical Outline](#)
- [Click on the Standard to find Sample Tasks](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Next Generation PARCC Resources](#)
- [Create Your Own Worksheets by Difficulty 1](#) (See Alg 1 and Alg 2 versions)
- [Kahn Academy - Rational Expressions](#)
- [Regents Exam Questions by topic](#)
- [Spark Notes - Rational Expressions](#)
- [Math Videos By Topic](#)
- [Math Assessment Project - Tasks by Standard](#)



Radical Expressions 1 Month

Cross-curricular Opportunities: Biology, Physics

Stage 1: Desired Results

Established Goals:

A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$

N.RN.1 – Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N.RN.2 – Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .

A.REI.2 – Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

F.BF.1 – Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F.BF.4 – Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Verify by composition that one function is the inverse of another. Read values of an inverse function from a graph or a table, given that the function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain.

F.JF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.JF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^x$, $y = (0.97)^x$, $y = (1.01)^{12x}$, $y = (1.2)^{x/10}$, and classify them as representing exponential growth or decay.

Transfer Goal:

Students will be able to independently use their learning to... Explain that functions are applicable to real world scenarios and they model patterns that can be related to other things.

Students will understand that:

- U1. Every function has an inverse.
- U2. A function and its inverse reverse the roles of the domain and range values
- U3. Radicals are real numbers and the same rules that apply to real numbers apply to radicals
- U4. Radical functions that have the same domain and that map to the real numbers can be added, subtracted, multiplied or divided (which may change the domain)
- U5. A function can be restricted by a domain and range and this can be seen in the graph.
- U6. Why real world problems are normally restricted to the first quadrant

Students will keep considering:

- Q1. In what situations is zero or a negative number an inappropriate answer to a problem?
- Q2. How do you know when to use a specific format?
- Q3. Does an inverse always have the same properties as the original?
- Q4. How can you tell when an answer makes sense?
- Q5. How can you relate the idea of like terms to like radical terms?

Students will know:

- K1. How to use properties of exponents with rational exponents
- K2. How to switch between root and exponent form and simplify
- K3. How to use and apply properties of radicals and exponents
- K4. How to create and use formulas involving exponents
- K5. How to solve and graph radical equations
- K6. How to analyze function operations and compositions
- K7. How to use and apply inverse functions

Students will be skilled at:

- A1. Finding the inverse of a simple (linear, quadratic, and cubic, radical functions).
- A2. Performing operations on radicals
- A3. Performing operations with functions including composition
- A4. Solving radical and rational exponential equations
- A5. Identifying an extraneous solutions
- A6. Solving systems graphically and algebraically
- A7. Finding and interpret domain and range within the context of a given problem
- A8. Writing a translated function given a parent function.
- A9. Determining the best model for a nonlinear set of data

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Radical Unit Test Bank](#)
- [Swing Time Project](#) - Student will construct pendulums using strings and weights and use your pendulums to investigate whether the length of the string or the amount of weight attached to a pendulum affects the time it takes the pendulum to make one full swing.
- [Numb3rs Activity - Burn Rate Episode - Radical Equations](#) - Students will use radical equations to solve problems about energy.
- [Group Activity](#) - Group activity requiring group members to work out problems individually to get answers. To check, groups add their final answers and check with teacher to make sure they are correct. A correct sum allows group to go on to next problems. Incorrect sums have students check each other's work to find the error.
- [Unit 3 Project - Walk This Way](#) - Students will examine the relationship between leg length and walking speed using radical equations
 - o [Unit 3 Project Rubric](#)
- [Unit 3 Review](#)
 - o [Unit 3 Review Answers](#)
- [Unit 3 Test](#)
 - o [Unit 3 Test Blue Print](#)
- Lesson Quizzes
 - o [6-1 LQ](#)
 - o [6-2 LQ](#)
 - o [6-3 LQ](#)
 - o [6-4 LQ](#)
 - o [6-5 LQ](#)
 - o [6-6 LQ](#)
 - o [6-7 LQ](#)
 - o [6-8 LQ](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

- Formal:
- Providing written/oral response to the EQs
 - Passing all quizzes on basic concepts in unit.
- Informal:
- Explaining how extraneous solutions occur and what they mean.
 - Troubleshooting examples explaining errors in reasoning and misuse of conventions or properties
 - Relating equations and graphs of radicals to those of quadratics.

Reference Materials

- [Radical Expression Topical Outline](#)
- [Click on the Standard to find Sample Tasks](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Next Generation PARCC Resources](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [Radical Resources](#)
- [Math Assessment Project - Tasks by Standard](#)



Exponentials and Logarithms

1 month

Cross-curricular Opportunities: Business, Biology, Social Studies

Stage 1: Desired Results

Established Goals:

A.SSE.3 - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15t$ can be rewritten as $(1.151/12)^{12t}$ $1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.CED.2 - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 - Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

F.IF.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts, intervals where the function is increasing, decreasing positive or negative, relative minimum and maximums, symmetries, end behavior and periodicity.

F.IF.7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8 - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^x$, $y = (0.97)^x$, $y = (1.01)^{12}$, $y = (1.2)/10$, and classify them as representing exponential growth or decay.

F.IF.9 - Compare properties to two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions). For examples, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.BF.1 - Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F.BF.4 - Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Verify by composition that one function is the inverse of another. Read values of an inverse function from a graph or a table, given that the function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain.

F.LE.4 - For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

F.LE.5 - Interpret the parameters in a linear or exponential function in terms of a context.

A.REI.11 - Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately using technology to graph the functions, make tables of values to find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions.

Transfer Goal:

Students will be able to independently use their learning to... Investigate important real-world phenomena from a graphical and an algebraic perspective relating to Logarithmic and exponential Equations.

Students will understand that:

- U1. Exponents are repeated multiplications
- U2. Log is an operation on a number, not an operation of two numbers like +, -, * and /
- U3. Logarithmic functions are the inverse of exponential functions
- U4. Exponential and logarithmic functions closely model many real-world phenomena
- U5. Inverse functions allow us to solve equations algebraically
- U6. Many of the previous ideas learned on other function families apply to exponential and logarithmic functions

Students will keep considering:

- Q1. Why do we need the logarithm function?
- Q2. How can we use what we know from past units to solve problem with these new functions?
- Q3. What real-world phenomena are modeled by exponential or logarithmic functions?
- Q4. How do exponents and logs compare to past material?
- Q5. What for the number e represent?
- Q6. Why does a logarithm with a base e have its own name?

Students will know:

- K1. How to identify key features of exponential functions and their graphs
- K2. How to identify key features of log functions and their graphs
- K3. How to compare exponential and log functions
- K4. How to use and apply the properties of logs
- K5. The relationships between exponential and log functions
- K6. How to evaluate and use different real-life applications of exponential and log functions
- K7. How to use the calculator/spreadsheets/tables to analyze exponential and log functions

Students will be skilled at:

- A1. Graph exponential and log functions
- A2. Evaluate exponential and log functions
- A3. Manipulate log expressions
- A4. Solve log and exponential equations
- A5. Apply compounded interest formulas
- A6. Use spreadsheet/table functions to evaluate and estimate conclusions

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Exponentials and Logarithms Unit Test Bank](#)
- Lesson Quizzes
 - o [7-1 LQ](#)
 - o [7-2 LQ](#)
 - o [7-3 LQ](#)
 - o [7-4 LQ](#)
 - o [7-5 LQ](#)
 - o [7-6 LQ](#)
- Unit 4 Quiz
- [Unit 4 Quiz Review](#)
 - o [Unit 4 Quiz Review Answers](#)
- [Unit 4 Blueprint](#)
- [Unit 4 Test](#)
- [Unit 4 Test Review](#)
 - o [Unit 4 Test Review Answers](#)
- [Zipf's Law Project](#) - Students will use Zipf's Law to model the relationship between population and rank of cities in a county
 - o [Zipf's Law Project Answers](#)
- [Performance Task - Predicting Population Changes](#)
- Banking Project - Students will choose a savings account that they determine to best meet their needs. Students will factor in interest rates, taxes, inflation and other characteristics of each account when they make their decision.
- Population Projects - Students will compare the population growth of two demographic groups. They will make predictions, and analyze how these changes make affect them individually, as a community, and as a country.

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal

- Students should also troubleshoot examples and explain misapplications of the conventions or properties
- Students can explain how to use logarithms
- Students can explain what exponential growth/decay is and how it differs from other types of growth/decay
- Students can explain the properties of inverse functions as related to exponential and logarithmic functions

Reference Materials

- [Many interactive Examples by Dan Meyer](#)
- [Textbook with content and examples](#)
- [Exponential Tasks](#)
- [Next Generation PARCC Resources](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [Math Assessment Project - Tasks by Standard](#)



Periodic Functions and Trigonometry 1 month

Cross-curricular Opportunities: Physics

Stage 1: Desired Results

Established Goals:

- F.IF.2** - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F.IF.7** - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F.TF.1** - Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F.TF.2** - Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F.TF.3** - Use special right triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$ and use the unit circle to express the values of sine, cosine and tangent for $\pi+x$, $\pi-x$ and $2\pi-x$ in terms of their values for x , where x is any real number.
- F.TF.4** - Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- F.TF.5** - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F.TF.6** - Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- F.TF.7** - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- F.TF.8** - Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Transfer Goal:

Students will be able to independently use their learning...Of periodic functions to analyze real life phenomena while keeping in mind the unit circle, its implications and its relationships.

Students will understand that:

- U1. Trig functions are ratios of sides of right triangles
- U2. Trig functions evaluated at a certain angle are constant, regardless of the size of the triangle
- U3. Trig identities allow us to express trig expressions in different, but equivalent ways
- U4. The Unit Circle allows us to evaluate trig functions at quadrantal angles
- U5. Reference angles allow us to evaluate trig functions at angles greater than 90 degrees
- U6. Radians are often a more appropriate way of describing angles
- U7. Tangent and Secant are named after their relationship to the unit circle
- U8. Changes to the algebraic equation of a function cause predictable changes to the function's graph
- U9. Basic properties of trig graphs can be linked to a deep understanding of the unit circle and fundamental trig identities
- U10. The symmetry of the unit circle allows trig equations to often have multiple answers
- U11. Trig equations can be treated like much more basic equations by using the concept of substitution

Students will keep considering:

- Q1. What are examples of things that happen periodically?
- Q2. How can trigonometry be used in the real world?
- Q3. How does changing one part of a triangle affect other parts of the triangle?
- Q4. How can radian measures be used in the real world?
- Q5. What is the value of knowing trigonometric identities?
- Q6. How does the unit circle enhance our understanding of trigonometric function?
- Q7. What is the benefit of having different units of measure?
- Q8. What is the benefit of having different representations?
- Q9. How can the unit circle begin to explain the graphs of sin, cos, tan, cot, sec and csc?

Students will know:

- K1. Basic trig identities
- K2. Pythagorean identities
- K3. Cofunction relationships
- K4. Basic trig functions of 30, 45 60 degree angles
- K5. The relationship between (cos, sin) and (x, y) on the unit circle
- K6. Sin, cos and tan of 30, 45, 60
- K7. Sin, cos and tan of quadrantal angles
- K8. Period formulas
- K9. Relationship between frequency, amplitude, volume and pitch
- K10. 5 Graphical Critical Points
- K11. Inverse trig functions
- K12. Laws of Sines and Cosines

Students will be skilled at:

- A1. Verify basic identities
- A2. Simplify trig expressions
- A3. Solve basic right triangle trig word problems
- A4. Convert between radians and degrees
- A5. Evaluate a trigonometric function for any multiple of 30, 45, 60 or 90
- A6. Find all six trig functions of an angle, given one
- A7. Fill out a unit circle (angles in radians, coordinates of points)
- A8. Graph all trigonometric functions and reciprocals by hand or on calculator
- A9. Graph arctrig waves
- A10. Solve linear trig equations
- A11. Solve trig equations with more than one function
- A12. Solve trig equations graphically

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Periodic Functions and Trig Unit Test Bank](#)
- [Chapter 13 Performance Tasks - Graphs and the Unit Circle](#) - Students will answer real world/graphical questions relating to the topics in Chapter 13
 - o [Chapter 13 Performance Tasks - Answers](#)
- [Unit 8 - PARCC PBA](#) - Scaffolded PBA focusing on Ferris wheel revolutions, graphs of trig functions and analysis
- [13-1 Notetaking Guide](#) - Student note taking guide with computer generated graphs for easier understanding
- [Discovering Radian Measure Lab](#)
- Lesson Quizzes

<ul style="list-style-type: none"> o 13-1 LQ o 13-2 LQ o 13-3 LQ o 13-4 LQ o 13-5 LQ o 13-6 LQ o 13-7 LQ o 13-8 LQ 	<ul style="list-style-type: none"> o 14-1 LQ o 14-2 LQ o 14-3 LQ o 14-4 LQ o 14-5 LQ
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- Customized Problem Solving Worksheets

<ul style="list-style-type: none"> o 13-1 o 13-2 o 13-3 o 13-4 • 13-1 to 13-4 Answers 	<ul style="list-style-type: none"> o 13-5 o 13-6 o 13-7 o 13-8 • 13-5 to 13-8 Answers 	<ul style="list-style-type: none"> o 14-1 o 14-1 Answers o 14-2 o 14-3 • 14-2 and 14-3 Answers
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- [Unit Circle](#) - Printable Unit circle with degrees, radians, sine and cosine values
- [Trigonometric Identities](#) - Basic Trig Identities reference sheet
- [Chapter 14 Performance Tasks](#) Trig Identities, Right/Non-Right Triangle Trig - Students will answer real world/graphical questions relating to the topics in Chapter 14

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

- Formal:
- Providing written/oral response to the EQs
 - Passing all quizzes on basic concepts in unit.
- Informal
- Students should also troubleshoot examples and explain misapplications of the conventions or properties
 - Students can explain the differences between Sine and Cosine Graphs
 - Students can create a unit circle from memory

Reference Materials

- [Periodic Functions and Trig Topical Outline](#)
- [Next Generation PARCC Resources](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [Kahn Academy - Modeling Periodic Functions](#)
- [Regents Prep - Arc Length and Radian Measure](#)
- [Regents Prep - Trig Graphing and Translation](#)
- [Trig translations - Lab, Notes and Practice](#)
- [Kahn Academy - Trig Graphs](#)
- [Law of Sines and COsines - Lab, Notes, Practice](#)
- [Another Ferris Wheel Scaffolded Example](#)
- [More complex Ferris Wheel Scaffolded Example](#)
- [Math Assessment Project - Tasks by Standard](#)



Data Analysis and Probability
1 Month

Teacher:

Cross-curricular Opportunities: Sciences, AP Statistics, Social Studies

Stage 1: Desired Results

Established Goals:

- S.CP.1** - Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- S.CP.2** - Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S.CP.3** - Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- S.CP.4** - Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
- S.CP.5** - Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
- S.CP.6** - Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
- S.CP.7** - Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- S.JC.1** - Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S.JC.2** - Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
- S.JC.3** - Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- S.JC.4** - Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S.JC.5** - Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S.JC.6** - Evaluate reports based on data.

Transfer Goal:

Students will be able to independently use their learning torationalize trends in sets of data and understand the meanings of the trends.

Students will understand that:

- U1. You can describe and compare sets of data using various statistical measures depending on what characteristics one would want to study
- U2. Standard deviation is a measure of how far the numbers in the set are away from the mean.
- U3. You can get good statistical information about a big group by studying a smaller set of the group.
- U4. Many common statistics gathered from samples in the natural world tend to have a normal distribution about their mean.
- U5. The chance of events happening is dependent on what information one wants to take from the situation.

Students will keep considering:

- Q1. What defines a good decision?
- Q2. How can I use probability to make wise decisions in my life?
- Q3. Why is the middle so important to a set of numbers?
- Q4. What is the best way to find out about a group of people?
- Q5. Why is the most common distribution of data called "normal"?

Students will know:

- K1. Logical terminology and different ways it is represented
- K2. Different types of events and their effect on each other, if any
- K3. How to organize data in different ways to best analyze it
- K4. How to calculate, recognize and analyze measures of central tendency, standard deviation and normal curves in a sample set of data.
- K5. Different types of sampling methods and survey techniques
- K6. How to calculate margin of error
- K7. How to evaluate measure of different sets of data

Students will be skilled at:

- A1. calculate measures of central tendency
- A2. draw and interpret box and whisker plots
- A3. find standard deviation and variance of a set of values
- A4. apply standard deviation and variance
- A5. Recognize the importance of margin of error
- A6. identify sampling methods
- A7. recognize bias in samples and surveys
- A8. use a normal distribution

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Data Analysis and Probability Unit Test Bank](#)
- [Probability Test](#)
- [Data Analysis Test](#)
- [Project - Who has the best Gas Prices](#) - Students will research and analyze gas prices across different regions of the country and draw conclusions from their findings
 - o [Project Gas Prices - Rubric](#)
- Survey Project - Students will create a survey based on information that is gathered from survey methods studied in class and infer information from the data
 - o [Survey Project Rubric](#)
- [Performance Task - Analyzing Data on Car Mileage](#) - Students will analysis a company's claims based on given data.
- [Mean Absolute Deviation Lab](#)
- Customized Worksheets by Lesson
 - o [11-1](#)
 - o [11-2](#)
 - o [11-3](#)
 - o [11-4](#)
 - o [11-1 to 11-4 Answers](#)
 - o [11-5](#)
 - o [11-6](#)
 - o [11-7](#)
 - o [11-8](#)
 - o [11-9](#)
 - o [11-10](#)
 - o [11-6 to 11-10 Answers](#)
- o Lesson Quizzes
 - o [11-1](#)
 - o [11-2](#)
 - o [11-3](#)
 - o [11-4](#)
 - o [11-5](#)
 - o [11-6](#)
 - o [11-7](#)
 - o [11-8](#)
 - o [11-9](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

- Formal:
- Providing written/oral response to the EQs
 - Passing all quizzes on basic concepts in unit.
- Informal
- Students should also troubleshoot examples and explain misapplications of the conventions or properties
 - Correctly decide which type of problem that is being solved BEFORE attempting to solve it

Reference Materials

- [Theme based units on compound probability](#)
- [Regents Prep - Pick Your Topic](#)
- [NJ Educator Exchange - Probability](#)
- [Kahn Academy - Combinatorics](#)
- [TI-Nspire Probability Lessons](#)
- [Next Generation PARCC Resources](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [NCTM Illuminations Activities - Probability and Data](#)
- [Common Core Standard Based Activities and tasks.](#)
- [Math Assessment Project - Tasks by Standard](#)



Sequences and Series

3 weeks

Teacher:

Cross-curricular Opportunities: Excel, Banking,

Stage 1: Desired Results

Established Goals:

A.SSE.4 - Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

F.IF.3 - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

F.BF.2 - Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Transfer Goal:

Students will be able to independently use their learning to...Analyze the behavior of patterned events.

Students will understand that:

- U1. Mathematical induction is a valid form of proof for sequence-type problems
- U2. Sequence and series formulas are common-sense shortcuts
- U3. Some infinite series converge to a single number
- U4. Sequences and series can be used to model real-life situations.
- U5. Sequences and series provide the foundation for upper level mathematics, especially calculus.
- U6. Sequences and series are a direct result of finding patterns.

Students will keep considering:

- Q1. Why do mathematical problems require proof?
- Q2. Why is it beneficial to generalize a pattern with an equation?
- Q3. How are sequences and series related?
- Q4. How can your calculator be used to help make things easier?
- Q5. What is the benefit of learning sequences and series?

Students will know:

- K1. How to interpret sequences and series, both arithmetic and geometric
- K2. Formulas for nth terms of arithmetic and geometric sequences
- K3. Formulas for finite arithmetic and geometric sequences
- K4. Formulas for infinite geometric series
- K5. The basics of Microsoft Excel

Students will be skilled at:

- A1. Finding nth terms of sequences
- A2. Evaluating series
- A3. Proving statements by mathematical induction
- A4. Evaluating summations
- A5. Setting up spreadsheets and use formulas in Excel

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Sequences and Series Unit Test Bank](#)
- [Sequences and Series Cheat Sheet](#) - Equations given to students
- [Geometric Sequences and Series Lab](#) - Students will use geometric shapes to model infinite series
- [Credit Card Project](#) - Students will use Excel to show the process of billing on a credit card using ideas from the sequences and series unit
- [Create your own Reality Series Project](#) - Students are responsible for creating and solving their own project involving series
- [Get the Picture Project](#) - Students will see how perspective affects perceived lengths and distances.
- [Sequences and Series Review](#)
 - o [Sequences and Series Review Answers](#)
- Lesson Quizzes
 - o [9-1 LQ](#)
 - o [9-2 LQ](#)
 - o [9-3 LQ](#)
 - o [9-4 LQ](#)
 - o [9-5 LQ](#)
- HW Pages
 - o [9-1 HW](#)
 - o [9-2 HW](#)
 - o [9-3 HW](#)
 - o [9-4 HW](#)
 - o [9-5 HW](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral responses to the EQs
- Passing all quizzes on basic concepts in the unit.

Informal

- Students should also troubleshoot examples and explain misapplications of the conventions or properties.
- Identifying which formula(s) to use for each problem given from reference sheet.

Reference Materials

- [Sequences and Series Topical Outline](#)
- [Student Note Taking Packet](#)
- [Teacher Notes](#)
- Student Note Taking Guides by Section
 - o [9-1 Worktext](#)
 - o [9-2 Worktext](#)
 - o [9-3 Worktext](#)
 - o [9-4 Worktext](#)
 - o [9-5 Worktext](#)
- [Next Generation PARCC Resources](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [Interactive Sequences and Series Applications](#)
- [Math Assessment Project - Tasks by Standard](#)



Conics
1 month

Teacher:

Cross-curricular Opportunities: Astronomy, Manufacturing, Art

Stage 1: Desired Results

Established Goals:

G.GPE.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.2 - Derive the equation of a parabola given a focus and directrix.

G.GPE.3 - Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

F.IF.8 - Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^x$, $y = (0.97)^x$, $y = (1.01)^{12}$, $y = (1.2)^x/10$, and classify them as representing exponential growth or decay.

A.REI.7 - Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

A.REI.1 - Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Transfer Goal:

Students will be able to independently use their learning to... realize that familiar rectangular equations in different ways.

Students will understand that:

- U1. Manipulation of Geometric shapes lead to Algebraic equations
- U2. Just like in Geometry, proof is necessary in Algebra for full understanding.
- U3. Using understanding from previous topics, topics can be expanded to prepared for more in depth courses in the future
- U4. It is beneficial to have an equation in x= form, not always y=

Students will keep considering:

- Q1. Does your perspective change shape of an object?
- Q2. Why must things be proven?
- Q3. What is the benefit of multiple representations?
- Q4. How can changing a small part of an equation largely affect the graph?
- Q5. What is the benefit of learning about conics?

Students will know:

- K1. How to derive the equation of a circle
- K2. How to derive the equation of a parabola
- K3. How to derive the equation of a hyperbola
- K4. Standard equations of conics given different parts of graph
- K5. How to find parts of graph given standard equations
- K6. How to solve systems involving conics and the graphical meanings
- K7. How to distinguish between the different types of conics.

Students will be skilled at:

- A1. Derive standard equations of conics
- A2. Graph conics
- A3. Identify conic section in non-formal situations
- A4. Use previous knowledge to apply concepts to conic equations
- A5. Solve systems involving one or two conic equations
- A6. Use previous knowledge of polynomials and rational equations to make sense on conics

Stage 2: Acceptable Evidence

Performance Task & Unit Assessments:

Students will show that they really understand by evidence of:

- [Conics Unit Test Bank](#)
- [Folding Conics Introductory Activity](#) - Students will fold wax paper to create the 4 Conic Shapes
- [Conics Equations Sheet](#)
- [Parametric Equations Task](#) - Students will use random data given to them to analyze the effects of velocity and angles on a thrown object in rectangular and parametric form
- [Polar Coordinate Activity](#) - Students will use polar equations to create aesthetically pleasing designs
- [Conics Quiz](#)
- [Conics Review](#)
 - o [Conics Review Answers](#)
- [Conics Test](#)
 - o [Conics Test Answer Sheet](#)
- [Conics HW sheet](#)
 - o [9-4 Answers](#)
 - o [9-5 Answers](#)
 - o [9-6 Answers](#)

Other Evidence:

Students will show that they have achieved Stage 1 goals by:

Formal:

- Providing written/oral response to the EQs
- Passing all quizzes on basic concepts in unit.

Informal

- Recognizing the type of a conic equation just by looking
- Students should also troubleshoot examples and explain misapplications of the conventions or properties
- Recognizing the benefit of Completing the square from Unit 1

Reference Materials

- [Student Note Taking Packet](#)
- [Teacher Notes](#)
- [Car Headlight Parabola Picture](#)
- [Conics Generated from a Cone - Picture](#)
- [Conic Tasks by standard](#)
- [History of Conics with Cross-Curricular References](#)
- [Interactive Applet](#)
- [Cutting Conics Activity with Questions](#)
- [Next Generation PARCC Resources](#)
- [PARCC EOY Assessment Evidence Table](#)
- [Regents Exam Questions by topic](#)
- [Math Videos By Topic](#)
- [Math Assessment Project - Tasks by Standard](#)