Verona Public School District
Curriculum Overview

AP Calculus AB/BC

Curriculum Committee Members:
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Curriculum Developed:
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Verona Public Schools
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Verona Public Schools Mission Statement:

The mission of the Verona Public Schools, the center of an engaged and supportive community, is to empower students to achieve their potential as active learners and productive citizens through rigorous curricula and meaningful, enriching experiences.

Course Description:

AP Calculus AB is roughly equivalent to a first semester college calculus course devoted to topics in differential and integral calculus. The AP course covers topics in these areas, including concepts and skills of limits, derivatives, definite integrals, and the Fundamental Theorem of Calculus. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations. Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations. Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

Prerequisite(s): Pre-Calculus/Trigonometry Honors or Pre-Calculus/Trigonometry Honors CP

Standard 8: Technology Standards	
8.1: Educational Technology: *All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.*	**8.2: Technology Education, Engineering, Design, and Computational Thinking - Programming:** *All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.*
x A. Technology Operations and Concepts B. Creativity and Innovation x C. Communication and Collaboration x D. Digital Citizenship x E. Research and Information Fluency x F. Critical thinking, problem solving, and decision making	A. The Nature of Technology: Creativity and Innovation x B. Technology and Society x C. Design D. Abilities for a Technological World x E. Computational Thinking: Programming

SEL Competencies and Career Ready Practices	
Social and Emotional Learning Core Competencies: *These competencies are identified as five interrelated sets of cognitive, affective, and behavioral capabilities*	**Career Ready Practices:** *These practices outline the skills that all individuals need to have to truly be adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.*
Self-awareness: The ability to accurately recognize one's emotions and thoughts and their influence on behavior. This includes accurately assessing one's strengths and limitations and possessing a well-grounded sense of confidence and optimism.	x CRP2. Apply appropriate academic and technical skills. x CRP9. Model integrity, ethical leadership, and effective management. x CRP10. Plan education and career paths aligned to personal goals.
Self-management: The ability to regulate one's emotions, thoughts, and behaviors effectively in different situations. This includes managing stress, controlling impulses, motivating oneself, and setting and working toward achieving personal and academic goals.	CRP3. Attend to personal health and financial well-being. x CRP6. Demonstrate creativity and innovation. x CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP11. Use technology to enhance productivity.
Social awareness: The ability to take the perspective of and empathize with others from diverse backgrounds and cultures, to understand social and ethical norms for behavior, and to recognize family, school, and community resources and supports.	x CRP1. Act as a responsible and contributing citizen and employee. x CRP9. Model integrity, ethical leadership, and effective management.
Relationship skills: The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. This includes communicating clearly, listening actively, cooperating, resisting inappropriate social pressure, negotiating conflict constructively, and seeking and offering help when needed.	x CRP4. Communicate clearly and effectively and with reason. x CRP9. Model integrity, ethical leadership, and effective management. CRP12. Work productively in teams while using cultural global competence.
Responsible decision making: The ability to make constructive and respectful choices about personal behavior and social interactions based on consideration of ethical standards, safety concerns, social norms, the realistic evaluation of consequences of various actions, and the well-being of self and others.	CRP5. Consider the environmental, social, and economic impact of decisions. x CRP7. Employ valid and reliable research strategies. x CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. x CRP9. Model integrity, ethical leadership, and effective management.

Standard 9: 21st Century Life and Careers		
9.1: Personal Financial Literacy: *This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.*	**9.2: Career Awareness, Exploration & Preparation:** *This standard outlines the importance of being knowledgeable about one's interests and talents, and being well informed about postsecondary and career options, career planning, and career requirements.*	**9.3: Career and Technical Education:** *This standard outlines what students should know and be able to do upon completion of a CTE Program of Study.*
A. Income and Careers B. Money Management C. Credit and Debt Management D. Planning, Saving, and Investing E. Becoming a Critical Consumer F. Civic Financial Responsibility G. Insuring and Protecting	A. Career Awareness (K-4) B. Career Exploration (5-8) x C. Career Preparation (9-12)	A. Agriculture, Food & Natural Res. B. Architecture & Construction C. Arts, A/V Technology & Comm. D. Business Management & Admin. E. Education & Training F. Finance G. Government & Public Admin. H. Health Science I. Hospital & Tourism J. Human Services K. Information Technology L. Law, Public, Safety, Corrections & Security M. Manufacturing N. Marketing x O. Science, Technology, Engineering & Math P. Transportation, Distribution & Log.

Course Materials	
Core Instructional Materials: *These are the board adopted and approved materials to support the curriculum, instruction, and assessment of this course.*	**Differentiated Resources**: *These are teacher and department found materials, and also approved support materials that facilitate differentiation of curriculum, instruction, and assessment of this course.*
• **Calculus (Larson), 7th Edition, Houghton Mifflin, 2002**	• AP Calculus Released Free Response Questions 1970-Present, Broadwin & Lenchner • AP Calculus Released Free Response Questions, 1998-Present, College Board • Delta Math • NJCTL • Khan Academy • Various online sources

Unit Title: Limits & Continuity	**Unit Duration:**
Stage 1: Desired Results	
Established Goals: **LO 1.1A(a):** Express limits symbolically using correct notation **LO 1.1A(b):** Interpret limits expressed symbolically **LO 1.1B:** Estimate limits of functions **LO 1.1C:** Determine limits of functions **LO 1.1D:** Deduce and interpret behavior of functions using limits **LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity **LO 1.2B:** Determine the applicability of important calculus theorems using continuity	
Transfer Goal: Students will be able to <u>independently</u> use their learning to understand and explain the limiting process as a tool for solving problems and a foundational element of the Derivative and the Definite Integral	
Students will understand that: EU 1.1: The concept of a limit can be used to understand the behavior of functions EU 1.2: Continuity is a key property of functions that is defined using limits	**Essential Questions:** EQ1: How can we describe the aesthetic properties real-world phenomena? EQ2: Why are functions that "behave well" or "behave badly" important to describing the world we live in? EQ3: How can we distinguish among infinite quantities or infinitesimal quantities? EQ4: How do limits extend and expand the power of geometry and algebra?
Students will know: EK 1.1A1: The meaning of a limit: Given a function f, the limit of f(x) as x approaches c is a read number R if f(x) can be made arbitrarily close to R be taking x sufficiently close to c (but not equal to c). EK 1.1A1: The relationship between meaning and notation of the limit of a function as x approaches c EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity and infinite limits EK 1.1A3: a limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded (blowup), if the function is oscillating near this value (infinite oscillation), or if the limit from the left does not equal the limit from the right (jump) EK 1,1B1: Numerical and graphical information can be used to estimate limits EK 1.1C1: Limits of sums, differences, products, quotients and composite functions can be found using the basic theorems of limits and algebraic rules EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem EK 1.1C3: Limits of the indeterminate forms 0/0 and Inf/Inf may be evaluated using L'Hospital's Rule EK 1.1D1: Asymptotic and unbounded behavior of functions can be explained and described using limits EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits EK 1.2A1: A function f is continuous at c if the limit at c exists and the limit equals the function value EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes EK 1.2B1: Continuity is an essential condition for theorems such as te Intermediate Value Theorem (IVT), Extreme Value Theorem (EVT), and the Mean Value Theorem (MVT)	**Students will be able to:** S1: Compute limits numerically S2: Compute limits graphically S3: Compute one-sided limits S4: Determine continuity of a function at a point S5: Use properties of limits to compute limits analytically S6: Determine the interval(s) on which a function is continuous S7: Apply the Intermediate Value Theorem to identify characteristics of a function S8: Identify and handle determinate and indeterminate forms S9: Compute limits analytically using algebraic and trigonometric techniques S10: Identify the zeroes and poles of rational functions and determine sketch them accordingly S11: Find limits of piecewise functions analytically S12: Identify and characterize discontinuities graphically and analytically S13: Compute infinite limits graphically S14: Compute infinite limits analytically across all function families
Stage 2: Acceptable Evidence	
Transfer Task: Slope and Area in Modeling the World with Polynomial Functions Students are given two functions: ● A function which models a quantity (count) measured over time ○ Students will plot the function and graphically estimate its slope at various times ○ Students will numerically estimate the slope at these same times ○ Students will analytically compute the slope, by extending the numerical approximation using the limiting process ○ Students will compare and contrast the results of these three approaches ● A function which models a rate process measured over time ○ Students will plot the function and use graphically estimation techniques to estimate the area over a time interval ○ Students will decompose the area into commonly known geometric figures to estimate the area over this same interval ○ Students will analytically compute the area, extending the numerical approximation via the limit process ○ Students will compare and contrast the results of these three approaches	

Unit Title: The Derivative	**Unit Duration:**

Stage 1: Desired Results

Established Goals:
LO 2.1A: Identify the derivative of a function as the limit of a difference quotient
LO 2.1B: Estimate Derivatives
LO 2.1C: Calculate Derivatives
LO 2.1D: Determine higher order derivatives
LO 2.2B: Determine Recognize the connection between differentiability and continuity

Transfer Goal:
Students will be able to <u>independently</u> use their learning to understand and explain rate of change concepts in mathematical models of real-world phenomena and generalize their knowledge of algebraic concepts such as slope through the Derivative.

Students will understand that:	**Essential Questions:**
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies EU 2.2: A function's derivative which is itself a function, can be used to understand the behavior of the function EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change	EQ1: Why are rate of change concepts of value to society? EQ2: How did the derivative extend physics and other sciences? EQ2: How does the derivative extend algebra?

Students will know:	**Students will be able to:**
EK 2.1A1: The difference quotient and alternative form the difference quotient express the average rate of change (AROC) of a function on an interval EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed as the limit of a difference quotient, provided that the limit exists. The formal and alternative definitions of the derivative can be denoted with f'(c) notation EK 2.1A3: The derivative of f is the function whose value (y-coordinate) at x is the limit of the difference quotient, provided that limit exists EK 2.1A4: Leibniz, Euler, and hybrid notation can be used to describe the derivative of a function. EK 2.1A5: The derivative can be represented graphically, numerically, analytically, and verbally. EK 2.1B1: The derivative at a point can be estimated from information given in tables or graphs EK 2.1C1: Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential and logarithmic functions EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules EK 2.1C4: The chain rule provides a way to differentiate composite functions EK 2.1C5: The chain rule is the basis for implicit differentiation EK 2.1C6: The chain rule can be used to find the derivative of an inverse function EK 2.1D1: Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f EK 2.1D2: Higher order derivatives can be represented with Leibniz, Euler or hybrid notations EK 2.2B1: A continuous function may fail to be differentiable at a point in its domain EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point	S1: Compute the average rate of change of a function on an interval numerically, graphically and analytically S2: Estimate the slope of a function at a point graphically or numerically S3: Compute the slope of a function at a point analytically S4: Determine the differentiability of a function at a point graphically or analytically S5: Determine the intervals on which a function is differentiable S6: Write the equation of a tangent line to a function at a point S7: Sketch the derivative a function given its graph S8: Use the power rule to differentiate algebraic functions S9: Use basic differentiation rules to differentiate transcendental functions S10: Use product and quotient rule to differentiate functions S11: Use chain rule to differentiate functions S12: Differentiate implicit functions S13: Compute higher-order derivatives

Stage 2: Acceptable Evidence

Transfer Task: Related Rates: Putting Laws of Math & Science into Motion

Students will create their own Related Rates problem which takes a law of Science or Geometry and create a dynamic version of that problem, illustrating it with a video presentation:

- Students will create a scenario which is governed by a law of mathematics (e.g. Pythagorean Theorem) or science (e.g. rectilinear motion, kinetic or potential energy, etc.)
- Students will solve and illustrate a static version of the problem
- Students will numerically measure rates which arise from the dynamic version of the problem at a specific instant in time using their video evidence
- Students will analytically derive the relationships between the rates in the dynamic version of the problem
- Students will validate the "dynamic version" of the law by substituting their measured rates
- Students will draw conclusions as to the validity of their derivation, taking into account any external or internal factors which may affect their results.

Unit Title: Applications of the Derivative	**Unit Duration:**

Stage 1: Desired Results

Established Goals:
LO 2.2A: Use derivatives to analyze properties of a function.
LO 2.3A: Interpret the meaning of a derivative within a problem
LO 2.3B: Solve problems involving the slope of a tangent line
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion and planar motion
LO 2.3D: Solve problems involving rates of change in applied contexts
LO 2.4A: Apply the Mean Value Theorem (MVT) to describe the behavior of a function over an interval

Transfer Goal:
Students will be able to independently use their learning to solve, understand and explain new classes of problems involving real-world phenomena, including optimization, function approximation, and establishing relationships between various rate processes within a system.

Students will understand that:
EU 2.2: A function's derivative which is itself a function, can be used to understand the behavior of the function
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change
EU 2.4: The Mean Value Theorem (MVT) connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval

Essential Questions:
EQ1: How can the derivative be used to describe the properties of real-world phenomena?
EQ2: How can the derivative be used to make our lives better?
EQ3: How can the derivative be used to save time and money in solving problems?
EQ4: How can the derivative be used to make geometry more valuable as a problem-solving tool?
EQ5: How much should we be willing to spend on the precision of knowledge?

Students will know:
EK 2.2A1: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection
EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations
EK 2.2A3: Key features of the graphs of f, f' and f'' are related to one another
EK 2.3C1: The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration
EK 2.3A1: The unit for f'(x) is the unit of f divided by the unit for x
EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable
EK 2.3B1: The derivative at a point is the slope of the line tangent to the graph at that point on the graph
EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency
EK 2.3C2: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to the other quantities whose rates of change are known
EK 2.3C3: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval
EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts

Students will be able to:
S1: Determine the intervals on which a function is increasing or decreasing
S2: Find relative extrema using the 1st Derivative Test
S3: Find critical number of a function
S4: Find absolute extrema of a function on a closed interval
S5: Determine intervals on which a function is concave up or concave down, graphically and analytically
S6: Find inflection points of a function graphically and analytically
S7: Find relative extrema using the 2nd Derivative Test
S8: Approximate a function value using a tangent line approximation
S9: Analyze the motion of an object along an axis
S10: Solve related rates problems
S11: Use the derivative to understand rates of change in an applied context
S12: Use the Mean Value Theorem to relate average rate of change to instantaneous rate of change
S13: Use L'Hospital's Rule to evaluate limits analytically

Stage 2: Acceptable Evidence

Transfer Task: Mathematical Modeling: Finding The "Best" Fit
Students will take a real-world situation and form a small dataset of ordered pairs of observations, manually create a least squares model, then validate that model using a computer or scientific calculator
- Students will record data observed from a simple experimental setup with two variables
- Students will create a plot of the data and manually create a best-fitting linear model, graphically measuring its intercept and slope
- Students will construct an error function (squared errors) analytically
- Students will graph the least squares function either using a 2-D graphing tool to graph contours of the error function, or a 3-D graphing tool such as Wolfram Alpha to graph the error function
- Students will find extrema by finding the critical numbers of the error function with respect to the slope and intercept of the linear model
- Students validate their analytical calculations by finding a line of best fit with the aid of a computational device such as Excel or the TI-Nspire

Unit Title: The Integral	**Unit Duration:**

Stage 1: Desired Results

Established Goals:
LO 3.1A: Recognize antiderivatives of basic functions
LO 3.2A(a): Interpret the definite integral as the limit of a Riemann sum
LO 3.2A(b): Express the limit of a Riemann sum in integral notation
LO 3.2B: Approximate a definite integral
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals
LO 3.2D: Evaluate an improper integral or show that an improper integral diverges (BC only)
LO 3.3A: Analyze functions defined by an integral
LO 3.3B(a): Calculate antiderivatives
LO 3.3B(b): Evaluate definite integrals

Transfer Goal:
Students will be able to independently use their learning to understand, explain, and solve problems involving real-world phenomena where rate processes are known.

Students will understand that:
EU 3.1: Antidifferentiation is the inverse process of differentiation
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies
EU 3.3: The Fundamental Theorem of Calculus (FTOC), which has two distinct formulations, connects differentiations and integration

Essential Questions:
EQ1: How can we count the uncountable?
EQ2: Why is counting important?
EQ3: How does the integral extend geometry?

Students will know:
EK 3.1A1: An antiderivative of a function f is a function g whose derivative f
EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives
EK 3.2A1: A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition
EK 3.2A2: The definite integral of a continuous function f over a closed interval, and denoted by definite integral notation is the limit of Riemann sums as the widths of the subintervals approach 0.
EK 3.2A3: The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral
EK 3.2B1: Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally
EK 3.2B2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions
EK 3.2C1: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area
EK 3.2C2: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals
EK 3.2D1: An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration (BC only)
EK 3.2D2: Improper integrals can be determined using limits of definite integrals (BC only)
EK 3.3A1: The definite integral can be used to define new functions such as accumulation functions
EK 3.3A2: If f is a continuous function on a closed interval, then the derivative of an accumulation function of f with respect to the upper limit of integrals equals f itself
EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f provide information about an accumulation function of f
EK 3.3B1: The function defined by an accumulation function of f is an antiderivative of f
EK 3.3B2: If f is continuous on a closed interval and F is the antiderivative of f, then the Fundamental Theorem of Calculus can be used to evaluate a definite integral analytically
EK 3.3B3: Indefinite integral notation can be used to describe indefinite integrals
EK 3.3B4: Many functions do not have closed form antiderivatives
EK 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, integration by parts (BC only), and nonrepeating linear partial fractions (BC only)

Students will be able to:
S1: Apply basic integration rules to evaluate indefinite integrals
S2: Find the constant of integration given an initial condition
S3: Find the net area between a curves and the x-axis using a definite integrals, graphically, numerically, or analytically
S4: Apply properties of definite integrals to compute them graphically or analytically
S5: Approximate the net area between a curve and the x-axis using a left, right, or midpoint Riemann sum, or a Trapezoid Sum
S6: Construct an accumulation function to represent the net area between a curve and the x-axis
S7: Use the Fundamental Theorem of Calculus to compute the net area between a curve and the x-axis
S8: Use the 2nd Fundamental Theorem of Calculus to differentiate accumulation functions
S9: Use accumulation functions to write the solutions to initial value problems
S10: Evaluate definite integrals of piecewise functions analytically
S11: Evaluate improper integrals (BC only)
S12: Evaluate indefinite and definite integrals using u-substitution
S13: Evaluate indefinite and definite integrals using integration by parts (BC only)
S14: Evaluate indefinite and definite integrals using partial fractions expansion (BC only)

Stage 2: Acceptable Evidence

Transfer Task: The Cafeteria Line - Analysis of a Closed System
Students will work in groups to analyze the behavior of a closed system with incoming and outgoing rate processes
- Students will work in groups of 3-4
- Students will measure the rate of arrival and rate of departure of students at the cafeteria line in increments of 2 minutes
- Students will develop a mathematical expression involving accumulation functions which expresses the queue length as a function of time
- Students will use the curve-fitting capabilities of the TI-Nspire to fit a higher-order polynomial model to the incoming and outgoing rate data
- Students will plot the rate functions over time
- Students will establish a four numerical approximations of the queue length as a function of time (Left, Right, and Midpoint Riemann, and Trapezoid)
- Students will compute an analytical solution using the fitted models for incoming and outgoing rate processes
- Students will develop an expression for average wait time and numerically approximate average wait time.
- Students will propose and debate solutions to reduce average wait time.

Unit Title: Applications of the Integral

Unit Duration:

Stage 1: Desired Results

Established Goals:

LO 2.3E: Verify solutions to differentiation equations
LO 2.3F: Estimate solutions to differentiation equations
LO 3.4A: Interpret the meaning of a definite integral within a problem
LO 3.4B: Apply definite integrals to problems involving the average value of a function
LO 3.4C: Apply definite integrals to problems involving motion
LO 3.4D: Apply definite integrals to problems involving area, volume, and length of a curve
LO 3.4E: Use the definite integral to solve problems in various contexts
LO 3.5A: Analyze differential equations to obtain general and specific solutions
LO 3.5B: Interpret, create, and solve differential equations from problems in context

Transfer Goal:

Students will be able to independently use their learning to express real-world problems in the language of differential equations and generalize their knowledge of geometric concepts through the Definite Integral.

Students will understand that:

EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation
EU 3.5 Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.

Essential Questions:

EQ1: How does the integral expand our knowledge of the world we live in?
EQ2: Why do we need differential equations to describe the universe?
EQ3: How can small changes in our world result in gigantic changes?

Students will know:

EK 2.3E1: Solutions to differential equations are functions or families of functions
EK 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation
EK 2.3F1: Slope Fields provide visual clues to the behavior of solutions to first order differential equations
EK 2.3F2: For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve (BC only)
EK 3.4A1: A function defined as an integral represents an accumulation of a rate of change
EK 3.4A2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval
EK 3.4A3: The limit of an approximating Riemann sum can be interpreted as a definite integral
EK 3.4B1: The average value of a function f over an interval is equal to the definite integral divided by the width of the interval
EK 3.4C1: For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time
EK 3.4D1: Areas of certain regions in the plane can be calculated with definite integrals
EK 3.4D2: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals
EK 3.4D3: The length of a planar curve defined by a function can be calculated using a definite integral
EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many applied contexts
EK 3.5A1: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, and logistic growth (BC only)
EK 3.5A2: Some differential equations can be solved by separation of variables
EK 3.5A3: Solutions to differential equations may be subject to domain restrictions
EK 3.5A4: Accumulation functions can be used to express particular solutions to initial value problems (IO-FTOC)
EK 3.5B1: The model for exponential growth and decay arises from the statement "The rate of change of a quantity is proportional to the size of the quantity"
EK 3.5B2: The model for logistic growth arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity"

Students will be able to:

S1: Compute an area between a curve and an axis
S2: Compute the area between two curves
S3: Compute the arc length of a curve
S4: Compute the net change of a quantity given a function describing its rate
S5: Analyze motion of an object along an axis
S6: Compute the average value of a function on an interval
S8: Compute the volume of a solid using disk method
S9: Compute the volume of a solid using washer method
S10: Compute the volume of a solid using volumes of common cross-section
S11: Compute areas and volume along the y-axis
S12: Verify the solution to an initial value problem
S13: Sketch a slope field describing an differential equation and the solution path given an initial condition
S14: Use Euler's Method to approximate the solution to an initial value problem (BC only)
S15: Use Separation of Variables to solve a separable differential equation or initial value problem

Stage 2: Acceptable Evidence

Transfer Task: Newton's Law Of Cooling

Students will conduct a small experiment to validate Newton's Law of Cooling, solving a differential equation, graphically, numerically, and analytically

- Students will use a microwave to heat a cup of water to the temperature of hot coffee/tea
- Students will plot the temperature versus time for four trials
- Students will analytically solve the differential equation for Newton's law of cooling
- Students will choose a constant which best fits their trial data
- Students will construct a spreadsheet which uses Euler's method to approximate solutions to each of the initial conditions of the trials
- For each trial, students will plot observed data versus the numerical approximation (Euler's Method) and the analytic solution
- Students will draw conclusions as to the validity of their numerical and analytical solutions, taking into account any external or internal factors which may affect their results.

Unit Title: Parametric & Polar Functions	Unit Duration:
Stage 1: Desired Results	
Established Goals: **LO 2.1C:** Calculate Derivatives **LO 2.3C:** Solve problems involving related rates, optimization, rectilinear motion and planar motion (BC only) **LO 3.4C:** Apply definite integrals to problems involving motion **LO 3.4D:** Apply definite integrals to problems involving area, volume, and length of a curve	
Transfer Goal: Students will be able to independently use their learning to extend their knowledge of motion in one dimension to motion in multiple dimensions and to understand and explain the power of reframing a mathematical problem in a way that makes problem-solving less difficult.	
Students will understand that: EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation	**Essential Questions:** EQ1: Why are parametric functions important in describing our world? EQ2: Why are polar functions important in describing our world? EQ3: Why have numerous mathematical representations of the world developed over time?
Students will know: EK 2.1C7: Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates EK 2.2A4: For a curve given by a polar equation, r=f(theta), derivatives of r, x, and y with respect to theta and first and second derivatives of y with respect to x can provide information about the curve. EK 2.3C4: Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions. EK 3.4C2: The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve, given by parametric or vector-valued functions EK 3.4D1: Areas of certain regions in the plane can be calculated with definite integrals. Areas bounded by polar curves can be calculated with definite integrals EK 3.4D3: The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many applied contexts	**Students will be able to:** S1: Compute velocity and acceleration vectors of a particle in planar motion S2: Compute the slope of a particle in planar motion S3: Find points of vertical or horizontal tangency of a particle in planar motion S4: Find the speed of a particle in planar motion S5: Find the change in position of a particle in planar motion S6: Find the total distance travelled by a particle in planar motion S7: Find the average speed of a particle in planar motion S8: Compute the slope of a polar curve at a point S9: Find horizontal and vertical points of tangency of a polar curve S10: Find the area enclosed by a polar curve S11: Find the area enclosed by two polar curves
Stage 2: Acceptable Evidence	
Transfer Task: Spirograph Students will work in groups to reverse-engineer the mathematics of curves drawn by the popular toy, Spirograph ● Students will explore the Spirograph drawing toy and come up with two original curves ● Students will make measurements of the distances of the tools which they specifically used to create their curves ● Students will develop analytical models (parametric representation) of their curves and may avail themselves to an encyclopedia of planar curve families to assist in developing their model ● If possible, students will translate their analytical models to implicit functions in rectangular coordinates (some curves might not permit such as representation) ● Students will plot their analytical models of their curves on the TI-Nspire calculator ● Students will use the Graph Trace feature to assist in superimposing the graph of the analytical models over their respective original Spirograph curves ● Students will analyze deviations between the graph of each analytical model versus its original curve ● Students will compute both analytically and graphically, the slope at particularly interesting points along the curve	

Unit Title: Power Series & Function Approximation	Unit Duration:

Stage 1: Desired Results

Established Goals:
LO 4.1A: Determine whether a series converges or diverges
LO 4.1B: Determine or estimate the sum of a series
LO 4.2A: Construct and use Taylor Polynomials
LO 4.2B: Write a power series representing a given function

Transfer Goal:
Students will be able to independently use their learning to understand and explain the deep relationship between transcendental functions and algebraic functions and to extend their knowledge of function approximation to develop better approximations of functions which nature does not directly reveal to us.

Students will understand that:
EU 4.1: The sum of an infinite number of real numbers may converge
EU 4.2: A function can be represented by an associated power series over the interval of convergence for the power series

Essential Questions:
EQ1: How have sequences played a crucial role in the progress of civilization in the last century?
EQ2: What is Discrete Mathematics and why do we need it?
EQ3: How are Discrete and Continuous mathematical concepts related?
EQ4: How are algebraic and transcendental functions related?

Students will know:
EK 4.1A1: The nth partial sum is defined as the sum of the first n terms of a sequence
EK 4.1A2: An infinite series of numbers converges to a real number S, if and only if the limit of its sequence of partial sums exists and equals S
EK 4.1A3: Common series of numbers include geometric series, the harmonic series, and p-series
EK 4.1A4: A series may be absolutely convergent, conditionally convergent, or divergent
EK 4.1A5: If a series converges absolutely, then it converges
EK 4.1A6: In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term tests, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test
EK 4.1B1: If a is the first term of a geometric series and r is the multiplier, then the convergence of the series depends upon the magnitude of the multiplier. The sum of a convergent geometric series is given by a formula.
EK 4.1B2: If an alternating series converges by the alternating series test (AST), then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series
EK 4.1B3: If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value
EK 4.2A1: The coefficient of the nth-degree Taylor polynomial centered at a is the nth derivative of the function evaluated at a, divided by n!
EK 4.2A2: Taylor polynomials for a function f centered at a can be used to approximate function values of f near a
EK 4.2A3: In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will converge to the original function over some interval
EK 4.2A4: The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function
EK 4.2A5: In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function
EK 4.2B1: A power series is a series in which the terms include successive powers of the difference between x and a number which represented the center of the series
EK 4.2B2: The Maclaurin series for sin(x), cos(x), and e^x provide the foundation for constructing the Maclaurin series for other functions
EK 4.2B3: The Maclaurin series for 1/(1-x) is a geometric series
EK 4.2B4: A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x)

Students will be able to:
S1: Determine the convergence/divergence of sequences
S2: Determine the convergence/divergence and sum of geometric series
S3: Apply the Nth Term Test to determine the divergence of a series
S4: Apply the Direct and Limit Comparison Tests to determine the convergence/divergence of a series
S5: Apply the Integral Test or p-series to determine the convergence/divergence of a series
S6: Apply the Ratio Test to determine the convergence/divergence of a series
S7: Apply the Alternating Series Test to determine the convergence of an alternating series
S8: Approximate an alternating series and find the approximation error bound using the Alternating Series Remainder
S9: Derive Power Series from geometric series and Maclaurin series for sine, cosine and exponential functions by using arithmetic operations, substitution, differentiation or integration
S10: Construct Taylor Polynomials for a function at a point
S11: Approximate the value of a function using a Taylor Polynomial and find the approximation error bound using either the Alternating Series Remainder or Lagrange Remainder

Stage 2: Acceptable Evidence

Transfer Task: BYO Scientific Calculator
Students will use construction Taylor Approximations to build a the transcendental function calculation capabilities of a basic scientific calculator
- Students will use Excel or Google Sheets to build a basic calculator for transcendental functions which is accurate to a specified precision
- Capabilities will include the six trigonometric functions, exponential and logarithmic functions
- Students may only use the five operations inherent in algebraic functions (addition, subtraction, multiplication, division, and exponentiation) in constructing their calculator
- Students will use the Lagrange or Alternating Series Error bounds to determine the number of series terms required to achieve the specified precision
- Students will generalized their calculator to include parameters for vertical and horizontal translation and dilation transformations
- Students will compare the results of their BYO calculator to a scientific calculator and reflect on the challenges of numerical approximation of transcendental functions