Applying Webb's Depth of Knowledge and NAEP Levels of Complexity in Mathematics

Marge Petit and Karin Hess

In order to define descriptors for cognitive demand to guide test item or assessment development, classification of items, and alignment to the states' Grade Level Expectations (GLEs), the Center for Assessment recommends drawing upon such work as Webb (2002), NAEP (2004) level of Complexities, and the implied cognitive demand in state GLEs for mathematics. These levels and descriptors can be used to guide item and overall test development, and establish the potential cognitive demand for assessment.

Descriptors of Levels for Mathematics (based on Webb, "Depth-of-Knowledge Levels for Four Content Areas," March 2002 and Webb, *Technical Issues in Large-Scale Assessment*, report published by CCSSO, December 2002
Below is a general definition for each Depth of Knowledge (DOK) Level. Table 1 (on the following page) contains mathematics descriptors for each level. Table 2 provides an example of a DOK ceiling level and other potential levels for assessment of a sample mathematics GLE.

Level 1 – Recall
This level involves the recall of information (fact, definition, term, or property), the use of a procedure, or applying an algorithm or formula. It also includes one-step word problems, and other specifications unique to content standards.

Level 2 –Skills and Concepts
The Skills and Concepts level involves demonstrating conceptual understanding through models and explanations, comparing and classifying information, estimating, and interpreting data from a simple graph. A Level 2 response requires students to make some decisions, such as how to approach the problem or activity.

Level 3 – Strategic Thinking
Strategic Thinking involves reasoning, planning, and using evidence to solve a problem or algorithm. Students would be asked at Level 3 to make and test conjectures, interpret information from a complex graph, solve complex problems, explain concepts, use concepts to solve non-routine problems, and provide mathematical justifications *when more than one response or approach is possible*.

Level 4 – Extended Thinking
Extended Thinking requires complex reasoning, planning, and thinking generally over extended periods of time (but not time spent only on repetitive tasks). At level 4, students may be asked to relate concepts to other content areas or to real-world applications in new situations. In mathematics, Level 4 Depth of Knowledge is not recommended by Webb to be assessed on the state grade level assessments, but should be assessed locally.

  khess@nciea.org

Table 1: *Math Descriptors* – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity (M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

Level 1 **Recall**	**Level 2** **Skills/Concepts**	**Level 3** **Strategic Thinking**	**Level 4** **Extended Thinking**
a. Recall, observe, or recognize a fact, definition, term, or property b. Apply/compute a well-known algorithm (e.g., sum, quotient) c. Apply a formula d. Determine the area or perimeter of rectangles or triangles given a drawing and labels e. Identify a plane or three dimensional figure f. Measure g. Perform a specified or routine procedure (e.g., apply rules for rounding) h. Evaluate an expression i. Solve a one-step word problem j. Retrieve information from a table or graph k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures l. Locate numbers on a number line, or points on a coordinate grid m. Solve linear equations n. Represent math relationships in words, pictures, or symbols o. Read, write, and compare decimals in scientific notation	a. Classify plane and three dimensional figures b. Interpret information from a simple graph c. Use models to represent mathematical concepts d. **Solve a routine problem** requiring multiple steps/decision points, or the application of multiple concepts e. Compare and/or contrast figures or statements f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones g. Provide justifications for steps in a solution process h. Extend a pattern i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps j. Translate between tables, graphs, words and symbolic notation k. Make direct translations between problem situations and symbolic notation l. Select a procedure according to criteria and perform it m. Specify and explain relationships between facts, terms, properties, or operations n. Compare, classify, organize, estimate, or order data	a) Interpret information from a complex graph b) Explain thinking when more than one response is possible c) Make and/or justify conjectures d) Use evidence to develop logical arguments for a concept e) Use concepts to solve non-routine problems f) Perform procedure with multiple steps and multiple decision points g) Generalize a pattern h) Describe, compare, and contrast solution methods i) Formulate a mathematical model for a complex situation j) Provide mathematical justifications k) Solve a multiple- step problem and provide support with a mathematical explanation that justifies the answer l) Solve 2-step linear equations/inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results m) Translate between a problem situation and symbolic notation that is not a direct translation n) Formulate an original problem, given a situation o) Analyze the similarities and differences between procedures p) Draw conclusion from observations or data, citing evidence	a) Relate mathematical concepts to other content areas b) Relate mathematical concepts to real-world applications in new situations c) Apply a mathematical model to illuminate a problem, situation d) Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results e) Design a mathematical model to inform and solve a practical or abstract situation f) Develop generalizations of the results obtained and the strategies used and apply them to new problem situations g) Apply one approach among many to solve problems h) Apply understanding in a novel way, providing an argument/justification for the application ***NOTE**: Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.*

  khess@nciea.org

Table 2: Sample DOK "ceiling" and potential Depth of Knowledge Levels for mathematics assessment

An important aspect to consider when designing grade level assessments is to use the highest Depth of Knowledge/Levels of Complexity demand implicit in a GLE as the "ceiling" for assessment, not the "target." The "ceiling" defines the highest levels of assessment of a GLE and the other (lower) levels with potential for assessment items. The "target" assumes that only the highest level is assessed.

Sample Mathematics GLE* for End of Grade 6	DOK Ceiling	Potential Levels for Assessment Up to DOK Ceiling
M–F&A–6–1 **Identifies, extends to specific cases, and generalizes a variety of patterns** represented in models, tables, graphs, sequences, or in problem situations; and writes a rule in words or^{SC} symbols for finding specific cases; and uses words or^{SC} symbols to express the rule/generalization of a linear relationship.	**3**	**1** Identifies a pattern **2** Extends a pattern to a specific case **3** Generalizes a pattern

*GLE NOTES: In this state example, the subscript "sc" indicates that students have a choice in how they complete the task (e.g., students can use words **or** symbols to express the rule).

Why is the distinction between "ceiling" and "target" important for test specifications and test development?

If one assessed only at the "target" level, all GLEs with a level 3 as their highest cognitive demand would only be assessed at level three. This would potentially have two negative impacts on the assessment: 1) The assessment as a whole would be too difficult; and 2) important information about student learning along the achievement continuum would be lost.

Specifying the DOK ceilings for each GLE and distribution of Depth of Knowledge/Levels of Complexity across the assessment will avoid these potential negative effects. The general protocol for this aspect is that a GLE should ***not*** be assessed above its "ceiling." To the extent possible, GLEs should be assessed at the "ceiling" and at least one level below the "ceiling" in order to provide additional diagnostic information to educators.

In April 2003, Norm Webb (email April 4, 2003) indicated that the current distribution of Depth of Knowledge used when applying the criterion in post hoc alignment analysis is at 50% of the items at a level 2 and above. However, he did not recommend a straight application of this distribution, but recommended that each state analyze their standards and related GLEs, and their vision to determine this distribution. (Source: *Vermont Revised Mathematics Test Specification, 2003*)

