

## AP Calculus Summer Assignment Part A

Read selected sections of [The Cartoon Guide To Calculus](http://www.veronaschools.org/Page/3128)

Which can be accessed at the following link:

<http://www.veronaschools.org/Page/3128>

Use the book and the web if necessary to find answer the questions.

### Chapter -1 : Speed, Velocity, Change

1. Calculus is the Mathematics of \_\_\_\_\_
2. What did Isaac Newton do in life besides help “discover” calculus?
3. What did Gottfried Leibniz do in life besides help “discover” calculus? How do you pronounce his name?
4. What’s the difference between *speed* and *velocity*?
5. What is *instantaneous velocity* and how did Newton and Leibniz approximate it?
6. What does the word *infinitesimal* mean?
7. Why did Newton call his method for exactly determining instantaneous velocity *fluxions* ?

### Chapter 0 : Meet The Functions

You can skip the bottom half of page 26, and all of 27-28, 52

8. How do you find the domain of a function?
9. Another way to write the absolute value function is  $|x| = \sqrt{x^2}$  . (Try it, it works). How is this related to the distance formula from geometry,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ?

10. List each of the “families” of elementary functions. For each family, list its most famous member and describe the kinds of real-world phenomena which can be explained or modeled by that family.

11. What are the various ways we can combine two functions to construct a new function?

12. What is a *one-to-one* function?

13. How are  $e^x$  and  $\ln x$  related?

14. How are the graphs of a function and its inverse related?

15. What does the input of a trig function represent? What does the output of a trig function represent?

16. What does the input of an inverse trig function represent? What does the output of an inverse trig function represent?

### **Chapter 1 : Limits**

You can skip pages 59-69,76

17. How would you explain the concept of a *limit* to your grandma?

18. Do limits represent x-coordinates or y-coordinates?

19. What is an *infinite limit*?

20. What is a *limit at infinity*?

21. Why doesn't  $\sin x$  have a limit as  $x$  approaches  $\infty$  ?

## **Chapter 2 : The Derivative**

You can skip pages 84-87, 94-100

22. If a function represents the position of an object, then the derivative of that function represents the object's \_\_\_\_\_

23. How is the definition of the derivative on page 83 related to the slope formula,  $m = \frac{y_2 - y_1}{x_2 - x_1}$  from algebra?

24. What is "Leibniz-style" notation for the derivative and why is it useful?

## **Chapter 3 : Chain, Chain, Chain**

You can skip this chapter

## **Chapter 4 : Using Derivatives, Part 1 : Related Rates**

Read only pages 117-118

25. Why do they call the problem described here a "Related Rates" problem?

26. Describe a real-life Related Rates problem you have encountered in the real world. Be sure to express the units of the two rates involved in the problem

## **Chapter 5 : Using Derivatives, Part 2 : Optimization**

Read only pages 125-128

27. Why is optimization important to living things? What kinds of things do we try to maximize? What kinds of things do we try to minimize?
  
28. If a function has a local maximum or minimum at a point, what is the value of the derivative at that point?

## **Chapter 6 : Acting Locally**

Read only pages 145 through the top third of 148, 153

29. What use is approximation? Why not just always calculate an exact answer?
  
  
  
  
  
  
  
  
  
  
30. Give four uses of the derivative with a short description of the importance of each

## **Chapter 7: The Mean Value Theorem**

You can skip this chapter

## **Chapter 8: Introducing Integration**

Read only pages 161-167

31. How are the meanings of and summation symbol  $\Sigma$  and the integration symbol  $\int dx$  similar? How are they different?

32. For objects moving in a straight line, the derivative helped us figure out instantaneous velocity when given the position of an object. What does integration help us figure out for objects moving in a straight line?

### **Chapter 9: Antiderivatives**

Read only pages 169-172

33. Why can a function have more than one antiderivative?

34. Why is integration sometimes “messier” or more difficult than differentiation (finding the derivative)?

### **Chapter 10: The Definite Integral**

Read only pages 177-top of 180, 182,185

35. Why do we need integration to compute area? Aren't the formulas we learned in geometry enough?

36. How is the limiting process necessary for us to understand what a definite integral is?

### **Chapter 11: Fundamentally.....**

Read only pages 187-188

37. The Fundamental Theorem of Calculus is the “mother of all area formulas”. Why?

## **Chapter 12: Shape-Shifting Integrals**

You can skip this chapter

## **Chapter 13: Using Integrals**

Read only pages 205-208, 220-226

38. Besides being useful for calculating areas and volumes, what else are definite integrals useful for?

39. What is a probability density function? What is a random variable?

## **Chapter 14: What's Next?**

Read pages 229-232

40. What makes a differential equation a differential equation?

AP Calculus Summer Assignment  
Part B

Demonstrate the following statements are true **without a calculator**

Do your work in your notebook you have purchased for class.

Note that the following problems are true statements and your job is to prove them using what you have learned in your prior math courses.

You should complete the problems “left-to-right” and not “right-to-left”

In these statements, the “given” information is leftmost in the statement and you are trying to prove the rightmost part of the statement

Here are some examples of what I mean when I say “left-to-right”

**Example Problem 1**

The solution of  $x + 2 = 8$  is  $x = 6$

- What I want you to do: solve the equation to show that the solution is  $x = 6$
- What I don't want you to do: substitute  $x = 6$  into the equation to verify that it is indeed the solution

**Example Problem 2**

$$(x + 3)(x - 2) = x^2 + x - 6$$

- What I want you to do: FOIL the left side of the equation
- What I don't want you to do: factor the right side of the equation

You will be able to solve many of these problems pretty quickly with your prior math knowledge. Some problems may require you to refresh your memory of a certain skill or technique. A select few problems may require a skill or technique which you may not have learned. If you are stuck on a problem and don't know how to start it, email me at [rwertz@veronaschools.org](mailto:rwertz@veronaschools.org) and I will send guidance to the entire class on how to attack the problem. If you find any errors in the problems, let me know and I will fix them.

1.  $\sqrt{50 + 288} = 13\sqrt{2}$
2.  $(\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{xy} + y$
3.  $\frac{3}{x-1} - \frac{3}{x+1} = \frac{6}{(x+1)(x-1)}$
4.  $\frac{2}{x+3} + \frac{2x+1}{x(x+3)^2} = \frac{2x^2+8x+1}{x(x+3)^2}$
5. The solutions of  $x^3 - 6x^2 - 27x = 0$  are  $x = -3, 0, 9$

6.  $\frac{x^2 y^{-1} 3^{-2}}{x^{-1} y^2 3^2} = \frac{x^3}{81 y^3}$
7.  $\frac{\frac{2}{x+2} - \frac{1}{x+2}}{\frac{3}{x}} = \frac{x}{3(x+2)}$
8. The solutions of  $-2x^2 - 8x + 1 = 0$  are  $x = -2 \pm \frac{3\sqrt{2}}{2}$
9.  $x + \sqrt{12x + 25} = -3$  has no solution
10. The solution of  $x + 2 - \sqrt{x} = 4$  is  $x = 4$
11. The solutions of  $x - 3 = -\sqrt{25 - 12x}$  are  $x = -8, 2$
12. If  $\frac{1}{y} = \frac{2}{x} - \frac{3}{5}$  then  $y = \frac{5x}{10 - 3x}$
13.  $\frac{3}{a} + \frac{3}{b} - \frac{6}{c} = \frac{3bc + 3ac - 6ab}{abc}$
14. The solution of  $\log_{10} x = 2$  is  $x = 100$
15. The solutions of  $(\log_{10} x)^2 + \log_{10} x^3 + 2 = 0$  are  $x = \frac{1}{100}, \frac{1}{10}$
16.  $\log_{10} 5 + \log_{10} 2 = 1$
17. The solutions of  $10^{2x} - 5 \cdot 10^x + 6 = 0$  are  $x = \log_{10} 3, \log_{10} 2$
18.  $(x^3)^2 = x^6$
19.  $\sqrt{x^3 \cdot x^2} = x^{\frac{5}{2}}$
20.  $x^2 - 4x + 5 = (x - 2)^2 + 1$
21.  $3x^2 + 5x - 2 = 3\left(x + \frac{5}{6}\right)^2 - \frac{49}{12}$
22.  $4x - x^2 + 5 = -(x - 2)^2 + 9$
23. The solution of  $|2x + 5| < 3$  is  $(-4, -1)$
24. The solution of  $|-3x + 1| > 5$  is  $\left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$
25.  $\log_{10} \left(\frac{x^2}{\sqrt{x-1}}\right) = 2 \log_{10} x - \frac{1}{2} \log_{10}(x - 1)$
26. The solution of  $\frac{1}{x} - \frac{1}{2(9-x)} = 0$  is  $x = 6$
27. The solutions of  $\frac{1}{x} - \frac{x}{4-x^2} = 0$  are  $x = \pm\sqrt{2}$
28. The solutions of  $-\frac{2}{x^2} + \frac{1}{2(x-3)^2} = 0$  are  $x = 2, 6$
29. The solution of  $\log_{10}(x - 2) = 1$  is  $x = 12$
30. The solution of  $\frac{2x(x^2-x)^{-\frac{1}{3}}(2x-1)}{3} + (x^2 - x)^{\frac{2}{3}} = 0$  is  $x = \frac{5}{7}$
31.  $(x^2 + 3x - 10)^{\frac{3}{2}} = (x - 2)^{\frac{3}{2}}(x + 5)^{\frac{3}{2}}$
32. If  $f(x) = x^{\frac{1}{2}} - x^{\frac{3}{2}}$  then  $f(9) = -24$



$$33. \frac{x^{-\frac{3}{4}}x^{\frac{3}{10}}}{\frac{1}{x^{12}}} = \frac{1}{x^{\frac{8}{15}}}$$

$$34. \frac{x^{\frac{3}{16}}x^{\frac{3}{4}}x^{-\frac{1}{6}}}{\frac{1}{x^8}} = x^{\frac{31}{48}}$$

For Problems 35-38, sketch the graph of the function and find the axis of symmetry, intercepts, and show justify the description of the graph

35.  $y = (x + 2)^2 + 1$  has an axis of symmetry of  $x = -2$ , a vertex at  $(-2,1)$  a y-intercept at  $(0,5)$ , no x-intercepts, and is a parabola opening upward

36.  $y = x^2 - 4x$  has an axis of symmetry of  $x = 2$ , a vertex at  $(2, -4)$ , a y-intercept at  $(0,0)$ , x-intercepts at  $x = 0$  and  $x = 4$ , and is a parabola opening upward

37.  $x = 4y^2 - 3y$  has an axis of symmetry of  $y = \frac{3}{8}$ , a vertex at  $(-\frac{9}{16}, \frac{3}{8})$ , y-intercepts at  $(0,0)$  and  $(0, \frac{3}{4})$ , an x-intercept at  $(0,0)$ , and is a parabola opening to the right

38.  $x^2 = 9 - y^2$  is a circle of radius 3, with a center at  $(0,0)$ , y-intercepts at  $(0, -3)$  and  $(0,3)$ , and x-intercepts at  $(-3,0)$  and  $(3,0)$

$$39. \cos\left(\tan^{-1}\frac{3}{4}\right) = \frac{7}{25}$$

$$40. 5x^2 - 13x + 6 = (x - 2)(5x - 3)$$

$$41. \text{The solutions of } 2x^2 + \frac{1}{3}x - \frac{1}{3} = 0 \text{ are } x = \frac{1}{3}, -\frac{1}{2}$$

$$42. \text{Show that } \frac{2x^3 - \frac{11}{3}x^2 = x + \frac{2}{3}}{x-2} = 2x^2 + \frac{1}{3}x - \frac{1}{3}$$

$$43. \text{Given that one root of } 2x^3 - \frac{11}{3}x^2 = x + \frac{2}{3} \text{ is } x = 2, \text{ show that } 2x^3 - \frac{11}{3}x^2 = x + \frac{2}{3} = (x - 2)(2x + 1)\left(x - \frac{1}{3}\right)$$

$$44. \text{The line through } (-4,1) \text{ with slope of } -\frac{1}{5} \text{ is } y = -\frac{1}{5}x + \frac{1}{5}$$

$$45. \text{The line with x-intercept of 3 and y-intercept of 5 is } y = -\frac{5}{3}x + 5$$

$$46. \text{The solutions to the system of equations } \begin{cases} y = -\frac{1}{5}x + \frac{1}{5} \\ x = y^2 - 6y + 1 \end{cases} \text{ are } (1,0) \text{ and } (-4,1)$$

$$47. \text{The line through } (-2,5) \text{ with an angle of inclination of } 45^\circ \text{ is } y = x + 7$$

48. The line through  $(-1,3)$  with an angle of inclination of  $120^\circ$  is  $y = -\sqrt{3}x - \sqrt{3} + 3$
49. The line through the points  $(1, -\frac{8}{3})$  and  $(-2, \frac{1}{3})$  is  $y = -x - \frac{5}{3}$
50.  $\frac{\frac{x^3}{3} - x^2 - 2x + \frac{8}{3}}{x^2 + x - 2} = \frac{1}{3}x - \frac{4}{3}$
51.  $(\sqrt{5}x + \sqrt{2}y)(\sqrt{5}x - \sqrt{2}y) = 5x^2 - 2y^2$
52.  $(\sqrt{2}x - 5)^2 = 2x^2 - 10\sqrt{2}x + 25$
53.  $(2x - y)^4 = 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$
54. If  $f(x) = x^2 - 4x + 1$  then  $f(\frac{\sqrt{3}+4}{2}) = -\frac{9}{4}$
55. The x-intercepts of  $y = x^3 - 16x$  are  $(0,0)$ ,  $(4,0)$ , and  $(-4,0)$
56. The x-intercepts of  $y = x^3 - x^2$  are  $(0,0)$  and  $(1,0)$
57. The solutions to the system of equations  $\begin{cases} y = -x^3 + \frac{3}{4}x^2 + \frac{3}{2}x + \frac{1}{2} \\ y = -x^2 + x + \frac{1}{2} \end{cases}$  are  $(0, \frac{1}{2})$ ,  $(2, -\frac{3}{2})$ , and  $(-\frac{1}{4}, \frac{3}{16})$
58. The solution to the system of equations  $\begin{cases} 9a + 3b + c = 2 \\ a + b + c = 4 \\ 6a + b = 0 \end{cases}$  is  $(\frac{1}{2}, -3, \frac{13}{2})$
59. The point of intersection of the lines  $y = 2x - 1$  and  $y = \frac{1}{3}x + 1$  is  $(\frac{6}{5}, \frac{7}{5})$
60. If the slope of a line through  $(-2,1)$  is 3, then the equation of the line through  $(-2,1)$  which is perpendicular to that line is  $y = -\frac{1}{3}x + \frac{1}{3}$
61. The solutions of  $x^4 - 6x^2 + 8 = 0$  are  $x = \pm 2$  and  $x = \pm\sqrt{2}$
62. The solutions to the system of equations  $\begin{cases} x^2 + 3y^2 = 13 \\ 2xy = -4 \end{cases}$  are  $(1, -2)$ ,  $(-1, 2)$ ,  $(2\sqrt{3}, -\frac{\sqrt{3}}{3})$ ,  $(-2\sqrt{3}, \frac{\sqrt{3}}{3})$
63. The solutions of  $(x + 3)^3(9x^2 + 6x + 33) - 3(3x^3 + 3x^2 + 33x + 25)(x + 3)^2 = 0$  are  $x = -3, 1$
64. The solutions of  $2(x + 3)^4(x - 1) - 4(x + 3)^3(x - 1)^2 = 0$  are  $x = -3, 1, 5$
65. If  $f(x) = \frac{x^2 - 2x}{x + 1}$  then  $f(-1 + \sqrt{3}) = 2\sqrt{3} - 4$
66.  $\frac{2x^2 + 3x - 2}{3x - 2} = \frac{2}{3}x + \frac{13}{9} + \frac{8}{9(3x - 2)}$

67.  $\frac{2x+1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$  then  $A = 7$  and  $B = -5$
68.  $\frac{2x-1}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$  then  $A = -\frac{1}{4}$ ,  $B = \frac{1}{4}$ , and  $C = \frac{3}{2}$
69.  $\frac{10x-20}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$  then  $A = -1$ ,  $B = 1$ , and  $C = 11$
70.  $\frac{2}{x-4} - \frac{3}{(x-4)^2} + \frac{2}{x-1} = \frac{4x^2-29x+43}{(x-1)(x-4)^2}$
71.  $\frac{x-1}{x^3-4x} - \frac{1}{x^2-4x+4} = \frac{-5x+2}{x(x+2)(x-2)^2}$
72.  $135^\circ = \frac{3\pi}{4}$  radians
73.  $5$  radians  $= \frac{900}{\pi}^\circ$
74. The length of an arc of a circle of radius 3 inches and central angle of  $30^\circ$  is  $\frac{\pi}{2}$  inches
75.  $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$
76.  $\cos \frac{29\pi}{6} = -\frac{\sqrt{3}}{2}$
77. If the angle of elevation to the top of a building from a point on the ground 40 feet from the foot of the building is  $60^\circ$  then the building must be  $40\sqrt{3}$  feet tall.
78. The solution of  $\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2\sqrt{2}}$  which lies in the 2<sup>nd</sup> quadrant of the unit circle is  $x = \frac{3\pi}{4}$
79. Given the identity  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$  we can find that  $\sin\left(-\frac{5\pi}{12}\right) = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$
80. Given the identity  $\cos 2x = \cos^2 x - \sin^2 x$ , we can convert the polar equation  $r = \cos 2\theta$  to Cartesian coordinates as  $(x^2 + y^2)^{\frac{3}{2}} = x^2 - y^2$
81. We can convert the Cartesian equation  $x = 3$  into polar coordinates as  $r = 3 \sec \theta$
82.  $y = \tan 2x$  has a period of  $\frac{\pi}{2}$  and vertical asymptotes at  $\frac{\pi}{4} \pm n\frac{\pi}{2}$
83.  $(x + 1 - \sqrt{2})(x - 1 + \sqrt{2}) = x^2 - 3 + 2\sqrt{2}$
84. If in  $\triangle ABC$ ,  $a = 5$ ,  $\sin A = 0.3$  and  $\sin B = 0.2$ , then  $b = \frac{10}{3}$
85. The solution of  $\sqrt{x^2 - 9} + 3 = x$  is  $x = 3$
86.  $\sqrt{24} - 6\sqrt{\frac{2}{3}} = 0$

87.  $\frac{1+\frac{1}{x}}{\frac{1}{1-x}} = \frac{1}{1-x}$  for  $x \neq 0, -1, 1$
88. If  $\sin x = \cos \frac{8\pi}{30}$  then  $x = \frac{7\pi}{30} + 2\pi n$  or  $x = \frac{23\pi}{30} + 2\pi n$  for any integer  $n$
89.  $\sin\left(\tan^{-1}\left(\frac{3}{4}\right)\right) = \frac{3}{5}$
90.  $\frac{\cos^2 x}{\sin x} + \sin x = \csc x$
91. If  $f(x) = 3x^0 - 2x^{-\frac{1}{3}}$  then  $f(8) = 2$
92.  $\sin(\pi + x) = -\sin x$
93. If  $\sin x = \frac{3}{5}$  then  $\sin 2x = \frac{24}{25}$  or  $-\frac{24}{25}$
94. The solutions to the system of equations  $\begin{cases} x^2 + y^2 - 4y - 5 = 0 \\ x - y - 1 = 0 \end{cases}$  are  $(0, -1)$  and  $(3, 2)$
95.  $\tan^{-1}(-1) = -\frac{\pi}{4}$
96. If  $x$  is a positive, acute angle and  $\cos x = \frac{\sqrt{21}}{5}$  then  $\sin x = \frac{2}{5}$
97. If in  $\triangle ABC$ ,  $b = 6$ ,  $c = 10$  and  $A = 30^\circ$  then the area of  $\triangle ABC$  is 15
98.  $\frac{2\sin^2 x}{\sin 2x} + \frac{1}{\tan x} = \sec x \csc x$
99. The solution to the system of equations  $\begin{cases} 2x + y + z = 2 \\ 4x - 2y - 3z = -2 \\ 8x + 3y + 2z = 5 \end{cases}$  is  $\left(\frac{1}{2}, -1, 2\right)$
100. If  $x$  and  $y$  are acute positive angles and  $\sin x = \frac{4}{5}$  and  $\cos y = \frac{8}{17}$  then  $\cos(x + y) = -\frac{36}{85}$
101. The smallest positive value of  $x$  which satisfies  $2\cos^2 x + \cos x - 1 = 0$  is  $x = \frac{\pi}{3}$
102.  $\frac{2(1+\cos x)}{\sin^2 x + \cos x + \cos^2 x} = 2$
103.  $\sin \frac{\pi}{6} + \cos \frac{\pi}{2} = \frac{1}{2}$
104.  $3\cos^2 x + \cos x - 2 = (\cos x + 1)(3\cos x - 2)$
105.  $\tan\left(\sin^{-1}\left(\frac{5}{13}\right)\right) = \frac{5}{12}$
106.  $\frac{\sqrt{50}}{3\sqrt{8}} = \frac{5}{6}$
107. If the length of an arc of a circle is 12 inches and has central angle of  $\frac{3}{2}$  radians then the radius of the circle is 8 inches

108. If the sides of a triangle are 2, 3, and 4 then the cosine of the largest angle in the triangle is  $-\frac{1}{4}$
109. If in  $\triangle ABC$   $A = C + 40^\circ$  and  $B = 2C$  then  $C = 35^\circ$
110. The solution of  $9^x = 27^2$  is  $x = 3$
111.  $\frac{5}{3-\sqrt{7}} = \frac{5\sqrt{7}+15}{2}$
112. If  $x$  is a positive acute angle,  $\cos x = \frac{1}{2}$  and  $y$  is a positive acute angle,  $\sin y = \frac{1}{2}$  then  $\sin(x + y) = 1$
113. If  $x$  is a positive acute angle,  $\cos x = \frac{1}{2}$  and  $y$  is a positive acute angle,  $\sin y = \frac{1}{2}$  then  $\cos(x - y) = \frac{\sqrt{3}}{2}$
114. If  $\sin(-x) = \frac{1}{3}$  then  $\sin x = -\frac{1}{3}$
115. If  $\cos(-x) = \frac{1}{3}$  then  $\cos x = \frac{1}{3}$
116.  $\frac{\tan x}{\sec x} = \sin x$
117. If the surface area of a sphere is  $36\pi$ , then its volume must also be  $36\pi$
118. If the perimeter of a rectangle is 20 inches and its diagonal is  $2\sqrt{17}$  inches, then its length must be 8 inches and its width must be 2 inches
119. The solutions of  $\cos 2x + \cos x = 0$  on  $[0, 2\pi]$  are  $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$
120. If in  $\triangle ABC$ ,  $C = 90^\circ$  and  $B = 45^\circ$  and  $b = \sqrt{3}$ , then the area of the triangle must be  $\frac{3}{2}$
121. If the lengths of two sides of a triangle are 7 and 10 and the cosine of the included angle is  $-\frac{1}{7}$ , then the third side of the triangle is 13
122. The solution of  $2 \sin^2 x = 1 + \sin x$  on  $[0, 2\pi]$  then  $x = \frac{7\pi}{6}, \frac{\pi}{2}, \frac{11\pi}{6}$
123. The period of  $y = \cos \frac{1}{2}x$  is  $4\pi$
124.  $2 \sin \frac{\pi}{6} - \tan^2 \frac{\pi}{3} = -2$
125.  $\cos \frac{\pi}{2} + \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
126. If a circle is centered at the origin has an arc of length 5 with a central angle of 1 radian, then the equation of the circle is  $x^2 + y^2 = 25$
127. The distance between  $(1, -2)$  and  $(6, 10)$  is 13
128. The midpoint of the segment joining  $(1, -2)$  and  $(6, 10)$  is  $(\frac{7}{2}, 4)$
129. The ellipse  $9x^2 - 18x + 4y^2 + 16y = 11$  has center  $(1, -2)$  and vertices  $(3, -2), (-1, -2), (1, 1), (1, -5)$

130. The equations of the asymptotes for the hyperbola  $\frac{x^2}{4} - y^2 = 1$  are  $y = \pm \frac{1}{2}x$
131. The equation of the line which is the perpendicular bisector of the line joining  $(1, -2)$  and  $(6, 10)$  is  $y = -\frac{5}{12}x + \frac{131}{24}$
132. If two points on a circle are  $(3, 2)$  and  $(6, -1)$  and the equation of a line passing through the center of the circle is  $y = 2x - 7$  then the equation of the circle is  $(x - 3)^2 + (y + 1)^2 = 9$
133. The coordinates of the points at which the parabola  $y = x^2$  intersects the unit circle are  $\left(\sqrt{\frac{\sqrt{5}-1}{2}}, \frac{\sqrt{5}-1}{2}\right), \left(-\sqrt{\frac{\sqrt{5}-1}{2}}, \frac{\sqrt{5}-1}{2}\right)$
134. An ellipse centered at the origin with vertices  $(4, 0), (0, -2), (-4, 0),$  and  $(0, 2)$  has the equation  $\frac{x^2}{16} + \frac{y^2}{4} = 1$
135. A hyperbola centered at the origin which intersects the  $y$ -axis at  $(0, 2)$  and  $(0, -2)$  and crosses  $\left(\frac{1}{2}\sqrt{5}, 3\right)$  has the equation  $\frac{-x^2}{1} + \frac{y^2}{4} = 1$
136. If the hypotenuse of a right triangle is 25 and one of its sides is 24, then the length of its other side is 7
137. If the vertices of a triangle are  $(1, -2), (4, 6),$  and  $(7, -2)$  then its area is 24
138. The solution of  $\log_2(x + 1) = 3$  is  $x = 7$
139.  $\triangle ABC$  is a right triangle with  $B = 90^\circ$  and  $A = 30^\circ$ . If a line is drawn from a point  $D$  on line  $AB$  to  $C$  such that  $\angle CDB = 60^\circ$  and  $AD = 20$ , then  $BC = 10\sqrt{3}$
140. Point  $A$  is outside a circle of radius  $r$ . A line is drawn from  $A$  to the center of the circle and another line through  $A$  tangent to the circle. The distance between  $A$  and the circle along the line through  $A$  to the center is 4. The distance between  $A$  and the point of tangency is 12. Show that the radius of the circle is 16
141. The circle with the equation  $x^2 - 6x + y^2 + 2y = -1$  has center  $(3, -1)$  and radius 3
142. If the equations of the top and bottom halves of a parabola are  $y = 2 + \sqrt{x + 4}$  and  $y = 2 - \sqrt{x + 4}$  respectively, then a single equation for the entire parabola is  $x = y^2 - 4y$
143. The solutions to the system of equations  $\begin{cases} x \sin y = 3 \\ x \cos y = \sqrt{3} \end{cases}$  are  $\left(2\sqrt{3}, \frac{\pi}{3} + 2\pi n\right)$  and  $\left(-2\sqrt{3}, \frac{4\pi}{3} + 2\pi n\right)$

144. If the vertices of a triangle are  $(0, -1)$ ,  $(0, 6)$ , and  $(4, 3)$  then its area is 14
145. If  $\tan A = \frac{1}{3}$  and  $\tan B = \frac{1}{4}$  then  $\tan(A + B) = \frac{7}{11}$
146. The solution of  $2^{x-1} = 2^x - 8$  is  $x = 4$
147. If a circle with center  $(2, 3)$  passes through  $(4, 1)$ , then the equation of the circle is  $(x - 2)^2 + (y - 3)^2 = 8$
148. If a parabola has the equation  $y = 2x^2 - 8x + 1$  then its axis of symmetry is  $x = 2$  and its vertex is  $(2, -7)$
149. If the interior angles of a triangle are  $x$ ,  $2x + 10$ , and  $\frac{1}{2}x + 30$  then the angles are  $40^\circ$ ,  $90^\circ$ , and  $50^\circ$
150. If a side of an equilateral triangle has length 2, then the area of the triangle is  $\sqrt{3}$
151. If the equal sides of an isosceles triangle are of length 4 and the angles opposite those sides are  $30^\circ$ , then the length of the third side is  $4\sqrt{3}$
152. If a line passes through  $(1, -3)$  and is parallel to the line  $4x + 3y = 1$  then the line is  $y = -\frac{4}{3}x - \frac{5}{3}$
153.  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$  for  $x \in (0, 1)$