take note — Key Concept: Vertex Principle of Linear Programming

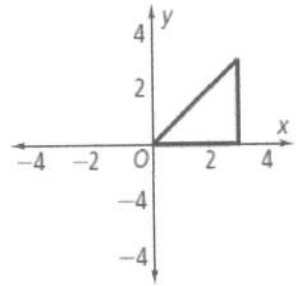
* If there is a maximum or a minimum value of a linear objective function, it occurs at one or ~~more vertices~~ of the feasible region.

The graph at the right shows a feasible region. Write the coordinates at which a maximum or minimum value of a linear objective function could occur.

vertices

(0 , 0) (3 , 0) (3 , 3)

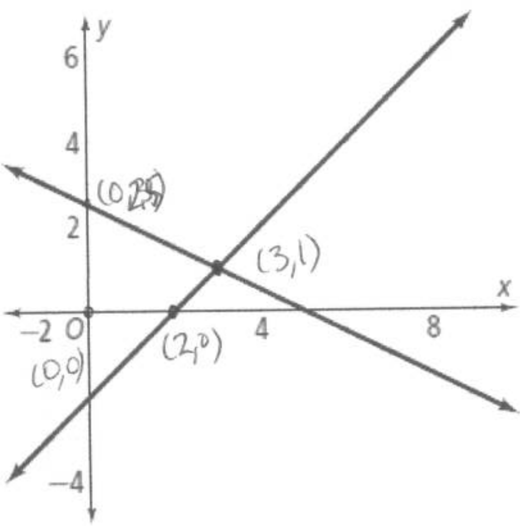
*DEPENDS ON OBJECTIVE FUNCTION



Example 1: Testing Vertices for Optimization

Got It? **Use the graph and the constraints below. What values of *x* and *y* in the feasible region maximize *P* for the objective function $P = x + 3y$?**

Label the vertices of the feasible region with their coordinates.



Constraints:

$$\begin{cases} x + 2y \le 5 \\ x - y \le 2 \\ x \ge 0, y \ge 0 \end{cases}$$

$y \le -\frac{1}{2}x + \frac{5}{2}$

$y \ge x - 2$

Find vertices:

(0,0) (0,2.5) (2,0) (3,1)

Evaluate function using each vertex:

$P = x + 3y$ Maximize

(0,0) $P = 0 + 3(0) = 0$

(0,2.5) $P = 0 + 3(2.5) = 7.5$ ✓

(2,0) $P = 2 + 3(0) = 2$

(3,1) $P = 3 + 3(1) = 6$

MAXIMIZE: FIND GREATEST VALUE

Example 2: Using Linear Programming to Maximize Profit

Got It? Business **You are screen-printing T-shirts and sweatshirts to sell at the Polk County Blues Festival and are working with the following constraints.**

- It takes 10 min to make a 1-color T-shirt.
- It takes 20 min to make a 3-color sweatshirt.
- You have 20 hours at most to make shirts.
- Supplies for a T-shirt cost $4.
- Supplies for a sweatshirt cost $20.
- You want to spend no more than $600 on supplies.
- You want to have at least 50 items to sell.

LIMITS ON TIME AND MONEY

The profit on a T-shirt is $6. The profit on a sweatshirt is $20. How many of each type of shirt should you make to maximize your profit?

Constraints:

$60 \times 20 = 1200$ min

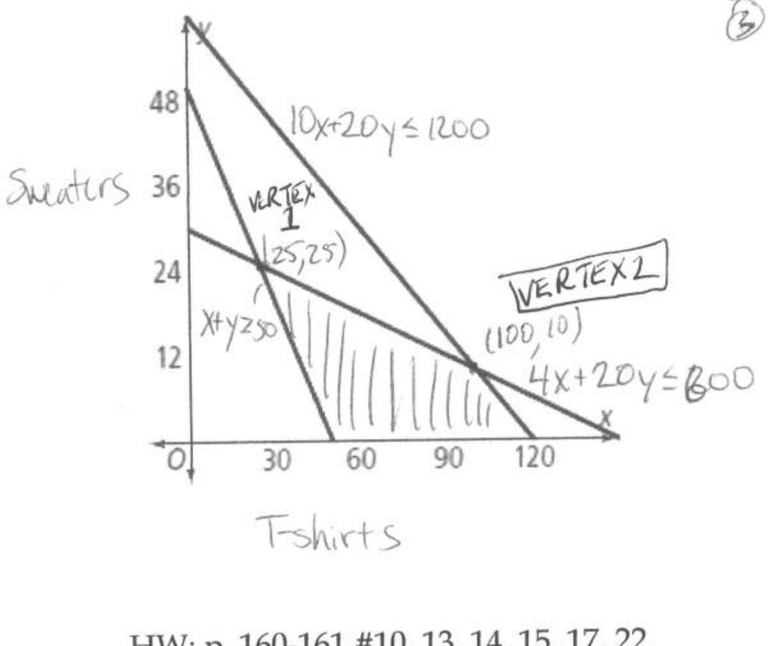
x = T-shirt

y = sweatshirt

$$\begin{cases} ① & 10x + 20y \le 1200 \\ ② & 4x + 20y \le 600 \\ ③ & x + y \ge 50 \\ ④ & x \ge 0 \\ ⑤ & y \ge 0 \end{cases}$$

$P = 6x + 20y$

Graph:



VERTEX 1

① $4x + 20y \le 600$

③ $x + y \ge 50$

$$\begin{aligned} 4x + 20y &= 600 \\ -4x - 4y &= -200 \\ \hline 16y &= 400 \\ y &= 25 \end{aligned}$$

$x + 25 = 50$

$x = 25$ $(25, 25)$

Vertex 2

$$\begin{aligned} ① \quad 10x + 20y &= 1200 \\ ② \quad -4x - 20y &= -600 \\ \hline 6x &= 600 \\ x &= 100 \end{aligned}$$

$4(100) + 20y = 600$

$20y = 200$

$y = 10$

$(100, 10)$

VERTICES: $(25,25)$ $(50,0)$ $(120,0)$ $(100,10)$

HW: p. 160-161 #10, 13, 14, 15, 17, 22

MAXIMIZE PROFIT

$P = 6x + 20y$

$(25,25) = 150 + 500 = \$650$

$(50,0) = 6 \cdot 50 + 0 = \$300$

$(120,0) = 6 \cdot 120 + 0 = \$720$

$(100,10) = 6 \cdot 100 + 20 \cdot 10 = 600 + 200 = \$800$ ✓

$\boxed{\text{100 T-shirts, 10 sweaters}}$

Name ANSWER KEY Class ______ Date ______

3-4 Think About a Plan

Linear Programming

Cooking Baking a tray of corn muffins takes 4 cups of milk and 3 cups of wheat flour. Baking a tray of bran muffins takes 2 cups of milk and 3 cups of wheat flour. A baker has 16 cups of milk and 15 cups of wheat flour. He makes $3 profit per tray of corn muffins and $2 profit per tray of bran muffins. How many trays of each type of muffin should the baker make to maximize his profit?

Understanding the Problem

1. Organize the information in a table.

	Corn Muffin Trays, *x*	Bran Muffin Trays, *y*	Total
Milk (cups)	$4x$	$2y$	16
Flour (cups)	$3x$	$3y$	15
Profit	$3x$	$2y$	P

2. What are the constraints and the objective function?

Constraints: $4x+2y \le 16$, $3x+3y \le 15$, $x \ge 0$, $y \ge 0$ Objective Function: $P = 3x+2y$

FIND INTERCEPTS

$4x+2y \le 16$: $(4,0)$, $(0,8)$

$3x+3y \le 15$: $(5,0)$, $(0,5)$

FIND INTERSECTION

$12x+6y=48$

$-6x-6y=-30$

$6x=18$

$x=3$

$3(3)+3y=15$

$3y=6$

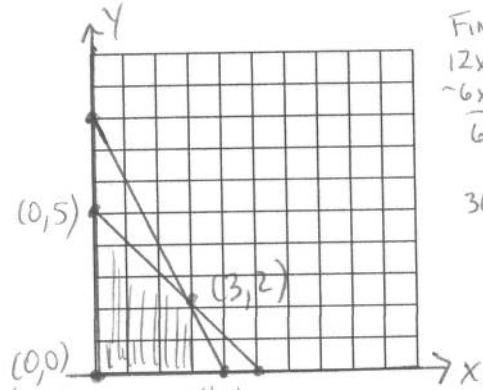
$y=2$

$(3,2)$

Planning the Solution

3. Graph the constraints on the grid at the right.

4. Label the vertices of the feasible region on your graph.



Getting an Answer

5. What is the value of the objective function at each vertex?

$P(0,5) = \$10$ $P(0,0) = 0$ $P(3,2) = \$13$ $P(4,0) = \$12$

6. At which vertex is the objective function maximized?

at (3,2)

7. How can you interpret the solution in the context of the problem?

make 3 corn muffins, 2 bran muffins to maximize profit